# OLD BABYLONIAN
# MATHEMATICAL PROCEDURE TEXTS
## A SELECTION OF "ALGEBRAIC" AND RELATED PROBLEMS WITH CONCISE ANALYSIS

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*Dedicato a Tommaso e Lucia*

## PRELIMINARY OBSERVATIONS

The following contains a selection of mathematical "procedure texts" – i.e., texts which first state a problem and then tell how to solve it – from the Old Babylonian period (2000 B.C. to 1600 B.C. – the mathematical texts belong to the second half of the period).

Most of the texts belong to the genre that is habitually regarded as *algebra* – as do indeed more than half of all Old Babylonian mathematical texts proper. One of the exceptions (VAT 8391 N° 3) shows how a first-degree problem is solved by non-algebraic means; the other two (IM 55357 and VAT 8512) demonstrate that the geometric operations used in the algebraic texts are also employed in the treatment of genuinely geometric problems (to such an extent indeed that it may be inappropriate to distinguish the two genres).

Traditionally, and since its existence was discovered around 1930, Babylonian "algebra" has been interpreted as a purely numerical technique, in the likeness of the algebra of the modern era; its terminology, which (to the extent that it could be interpreted in ordinary language) suggested a geometrical reading, was taken to be a set of frozen metaphors (as is our "square" of a number). The detailed reasons underlying this received interpretation, as well as the arguments that it does not hold water, I have presented in a number of publications – in particular [Høyrup 1990a; Høyrup 1991]. The main point is that the numerical interpretation makes sense of (most of) the numbers that occur in the texts. However, it leaves many phrases and terms as inexplicable; moreover, it is unable to explain why the texts distinguish sharply between two different operations which arithmetically seen are one and the same "addition"; between two different "subtractions" – and between no less than four different "multiplications". Finally, it often makes it difficult to understand the order in which operations are performed.

The alternative is a reading which in as far as it is possible takes its bearings from the original wording, phrasing and ordering rather than from the patterns of thought of modern mathematics. This reading I shall present in the following, introducing and explaining the techniques and concepts as they occur in the texts[1]. For this purpose I shall use “conformal” standard translations, i.e., translations which render each single Babylonian term by the same English term (when possible with a roughly similar range of connotations), related terms by similarly related translations, and distinct terms by distinct translations; which, furthermore, follows the original grammar, word order and phrase structure unless the result becomes completely unreadable, and which in general tries to tell nothing beyond what is in the original text (evidently, definite and indefinite articles have to be inserted; on the other hand, the rich verbal system of Akkadian can only be rendered by means of circumlocutions and similar stratagems).

The texts were originally written in cuneiform on clay tablets. Their basic language is Babylonian, one of the two main dialects of Akkadian. To a varying degree, however, the texts contain terms of Sumerian origin. In some cases, these are to be understood as logograms (word signs), abbreviated writings for Akkadian words (thus ZI stands for a variety of conjugated forms of the Akkadian verb *nasāḫum*); in others, however, they were read in Sumerian (and eventually borrowed into Akkadian as loanwords). Logograms for Akkadian are translated as are the corresponding Akkadian terms (in the grammatical form which is to be expected from the context[2]); authentic Sumerian terms are given a translation of their own[3].

Numbers were written in a floating-point place-value system with base

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[1] For convenience, an appendix (p. 52) contains a recapitulation and the Akkadian and Sumerographic equivalents of all translated terms, while an index (p. 56) locates the passages where the single operations and terms are introduced and discussed.

It should be emphasized that the explanations that accompany the texts do not present the full evidence for the interpretation. This requires contrastive comparison of a large number of texts, in many cases even analysis of all occurrences of a term within the complete corpus – see [Høyrup 1990a] and [Høyrup 1993b].

[2] In a few texts, however, the Sumerograms are to be read as infinite lexical forms, not as the finite verbs that would fit grammatically.

[3] In the lines containing the original text in transliteration, syllabic Akkadian is written in *italics*, whereas logograms, genuine Sumerian terms and signs of unidentified reading occur as SMALL CAPS.

60; the number which we transliterate 4,46,40 may thus stand for $4 \cdot 60^2+46 \cdot 60+40$ as well as $4 \cdot 60+46+40 \cdot 60^{-1}$ or $4+46 \cdot 60^{-1}+40 \cdot 60^{-2}$ (etc.). In the translations, a generalized degree-minute-second system is used, in which ´, ´´, etc. indicate decreasing and `, ``, etc. increasing sexagesimal order of magnitude; 4`46°40´ thus stands for $4 \cdot 60+46+40 \cdot 60^{-1}$ (in some cases, this absolute order of magnitude can be determined from the calculations; in others, it is fixed arbitrarily or from the topic dealt with)[4]. In certain cases, numbers are written as number words, in which case they are translated correspondingly. Similarly, the fractions $^1/_2$, $^1/_3$, and $^2/_3$ possess their own signs, which are transliterated and translated as ordinary fractions.

Indications of damages to the text (etc.) are only given in the lines in original language, unless the formulation is not firmly established from parallel passages; in such cases, the conjectural restitution is indicated as ¿…?.

## I. FIRST-DEGREE PROBLEMS

### *TMS XVI, Nº 1*[5]

**1.** The 4th of the width from the length and the width to tear out, 45´. You, 45´
[4-*at* SAG *i-na*] UŠ *ù* SAG ZI 45 ZA.E 45

**2.** to 4 raise, 3 you see. 3, what is that? 4 and 1 posit,
[*a-na* 4 *i-ší* 3 *ta*]-*mar* 3 *mi-nu šu-ma* 4 *ù* 1 GAR

4 This system – all-dominating in the mathematical texts and to all evidence originally introduced as a tool for intermediate technical calculations, similar to the equally floating-point-based slide rule – was not in general use in practical contexts, where the order of magnitude had to be made explicit; economic and similar texts use other notations that leave no doubt whether a debt is 300, 5 or $^1/_{12}$ *šekel*. This should go by itself but is often forgotten when histories of mathematics present "the Babylonian numerals".

5 Transliteration [TMS, 91f] (the translation and commentary of this edition are mistaken and highly misleading). Corrections, translation and analysis [Høyrup 1990a: 299–305].

**3.** 50´ and 5´, to tear out, posit. 5´ to 4 raise, 1 width. 20´ to 4 raise,
[50 *ù*] 5 ZI ⌈GAR⌉ 5 *a-na* 4 *i-ší* 1 SAG 20 *a-na* 4 *i-ší*

**4.** 1°20´ you see, 4 widths. 30´ to 4 raise, 2 you see, 4 lengths. 20´, 1 width to tear out,
1,20 *ta-*⟨*mar*⟩ 4 SAG 30 *a-na* 4 *i-ší* 2 *ta-*⟨*mar*⟩ 4 UŠ 20 1 SAG ZI

**5.** from 1°20´, 4 widths, tear out, 1 you see. 2, the lengths, and 1, 3 widths, accumulate, 3 you see.
*i-na* 1,20 4 SAG ZI 1 *ta-mar* 2 UŠ *ù* 1 3 SAG UL.GAR 3 *ta-mar*

**6.** The IGI of 4 detach, 15´ you see. 15´ to 2, lengths, raise, 30´ you see, 30´ the length.
IGI 4 *pu-*[*tú-ú*]*r* 15 *ta-mar* 15 *a-na* 2 UŠ *i-ší* 3[0] *ta-*⟨*mar*⟩ 30 UŠ

**7.** 15´ to 1 raise, 15´ the contribution of the width. 30´ and 15´ retain.
15 *a-na* 1 *i-ší* [1]5 *ma-na-at* SAG 30 *ù* 15 *ki-il*

**8.** Since “The 4th of the width, to tear out”, he has said, from 4, 1 tear out, 3 you see.
*aš-šum* 4-*at* SAG *na-sà-ḫu qa-bu-ku i-na* 4 1 ZI 3 *ta-mar*

**9.** The IGI of 4 detach, 15´ you see, 15´ to 3 raise, 45´ you see, 45´ as much as (there is) of widths.
IGI 4 *pu-*⟨*tú-úr*⟩ 15 *ta-mar* 15 *a-na* 3 *i-ší* 45 *ta-*⟨*mar*⟩ 45 *ki-ma* [SAG]

**10.** 1 as much as (there is) of lengths posit. 20, the true width take, 20 to 1´ raise, 20´ you see.
1 *ki-ma* UŠ GAR 20 GI.NA SAG *le-qé* 20 *a-na* 1 *i-ší* 20 *ta-mar*

**11.** 20´ to 45´ raise, 15´ you see. 15´ from $30^{15}$ tear out,
20 *a-na* 45 *i-ší* 15 *ta-mar* 15 *i-na* $30^{15´}$ [ZI]

**12.** 30´ you see, 30´ the length.
30 *ta-mar* 30 UŠ

The present text is highly untypical *as a text*. It does not solve a problem but explains the meaning of the steps by which an equation is transformed, and thus makes explicit what is implicit in most of the material at our disposal. This character may have to do with the origin of the text: it was written in Susa, a peripheral area, toward the very end of the Old Babylonian period, and teachers from a peripheral school may have felt the need for written instructions where those from the core could rely on a more firmly established tradition of oral explanations. But there is no reason to believe that the written explanation of our Susa text deviates from the oral expositions given elsewhere[6].

The problem (which is really *an equation*) deals with the length ($l$) and the width ($w$) of a rectangle; in the present case, however, this concrete meaning is relatively unimportant.

In line 1, we are told (in symbolic translation) that

$$(l+w)-{}^{1}/_{4}\,w = 45' \,.$$

Already here we encounter the problem of different "additions" and different "subtractions". One additive operation ("accumulating" *a* and *b*, represented here by a mere "and") is a real addition which absorbs the addends in a common sum (at times spoken of in the plural, as "the things accumulated", at times in an apparent singular, "the accumulation"; it may be used for the purely arithmetical addition of entities of different kinds (e.g., lengths and areas), provided that they possess a measuring number. The other ("appending" *a* to *B* – absent from the present text) is a concretely meaningful operation, in which *B* so to speak conserves its identity and (if the operation is geometrical) stays in place while increasing in size.

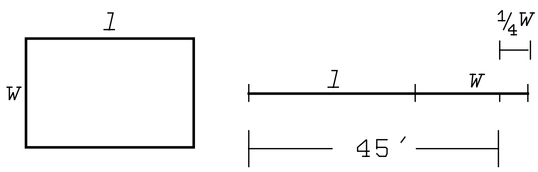


**Figure 1**.

The subtraction of line 1 (to "tear out" *a* from *B*) is also an "identity-conserving" operation, and can only be used when a concrete removal of a portion of *B* is dealt with. The other subtractive operation, the observation that "*A* goes *d* beyond *B*" allows us to find the difference *d* between magnitudes one of which cannot be considered part of the other, and where removal is thus excluded. This difference may be spoken of as "so much as *A* goes beyond *B*" or simply as the "going-beyond".

Apart from what is translated into symbols, line 1 thus tells us that $l$ and $w$ are aggregated on an equal footing – along a common line, we may imagine (see Figure 1) – after which ${}^{1}/_{4}$ of the width can really be

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[6] The interpretation of a passage in TMS IX as evidence of specific Susian methods (underscored in the preface to the volume, and often quoted in the secondary literature) relies on a double misunderstanding – cf. [Høyrup 1990a: 326].

*removed*; what remains equals 45´. We observe that the text, though dealing with geometrical line segment, regards these as *measurable*, and indeed measured. This is a general characteristic of the Old Babylonian "algebra" texts – the geometrical entities they speak of are always thought of as possessing a measuring number, which is often used as an identifying name (occasionally even when this number is not considered as given, cf. p. 43).

The number 45´, the student is told (lines 1–2), is to be multiplied by 4. "Raising", indeed, is one of the four "multiplications". Its original use will have been in the computation of volumes, where an area *A* provided with an implicit standard height 1 is "raised" to the real height *h* (see [Høyrup 1992: 351f]). From there, the term was transferred to other cases where a computation involved some consideration of proportionality – ultimately we may think of it as "computation of a concrete magnitude through multiplication". This multiplication yields 3, the meaning of which is asked for.

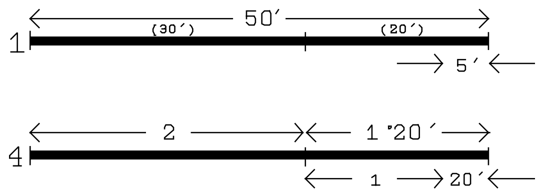


**Figure 2**.

The explanation shows that the values of *l* (30´) and *w* (20´) are presupposed. At first one is to "posit" 4 and 1 (for the multiplied and the original equation). "Positing" appears to designate various kinds of material recording – "putting down" in a calculation scheme, writing the value of a length or an area into a drawing, etc. We may imagine something like Figure 2 (without believing too firmly in the exact details of the representation). Next it is explained that 4·5´ yields 20´, one width, that 4·30´ yields 4 lengths, etc.

In line 6 we encounter a new operation. The IGI of the number *n* is its reciprocal as listed in the table of reciprocals, and "to detach" it means to look it up in this table (originally probably to detach one part from a bundle consisting of *n* parts). Line 6 thus multiplies the equation by $^{1}/_{4}$ and now identifies the single contributions as multiples of *l* and *w*. Of particular interest is the explicit determination of the coefficients, "as much as (there is)" of lengths and widths (1 and 45´ = $^{3}/_{4}$, respectively – the latter determined from an argument of the type "single false position" in lines 8–9).

The distinction between the "width" and the "true width" in line 10 probably means that an (imaginary) *real* rectangular field is *represented* by another rectangle – as suggested in the translation, the former *may* have had the dimensions 30×20 and the latter the more manipulable[7] dimensions 30´×20´.

All in all, as we see, the text is a highly pedagogical exposition, moving back and forth between the various levels so as to create full understanding of their mutual connections; but no attempt is made to achieve anything like a deductive structure.

### *TMS VII, N° 2[8]*

**17.** The 4th of the width to the length I have appended, its 7th
4-*at* SAG *a-na* UŠ DAḪ 7-*ti*[-*šu*]

**18.** until 11 I have gone, over the accumulation
*a-di* 11 *al-li-ik* UGU [UL.GAR]

**19.** of length and width 5´ it goes beyond. You, 4 posit;
UŠ *ù* SAG 5 DIRIG ZA.E [4 GAR]

**20.** 7 posit; 11 posit; and 5´ posit.
7 GAR 11 GAR *ù* 5 GAR

**21.** 5´ to 7 raise, 35´ you see.
5 *a-na* 7 *i-ší* 3[5 *ta-mar*]

**22.** 30´ and 5´ posit. 5´ to 11 raise, 55´ you see.
30 *ù* 5 GAR 5 *a-na* 1[1 *i-ší* 55 *ta-mar*]

**23.** 30´, 20´ and 5´, to tear out, posit. 5´ to 4
30 20 *ù* 5 ZI GAR 5 [*a-n*]*a* 4

**24.** raise, 20´ you see, 20´ the width. 30´ to 4 raise,
*i-ši* 20 *ta*-⟨*mar*⟩ 20 SAG 30 *a-na* 4 *i-ší-ma*

**25.** 2 you see, 2, lengths. 20´ from 20´ tear out.
2 *ta-mar* 2 UŠ 20 *i-na* 20 ZI

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7 The unit of horizontal measures is the NINDAN or "rod", roughly equal to 6 m (whereas vertical distances are measured in KÙŠ or "cubit", 12 KÙŠ = 1 NINDAN). A rectangle 30×20 is thus roughly 180 m by 120 m, much too large to be traced in the school yard (or whatever "blackboard" was used).

8 Transliteration [TMS, 54f] (the translation and commentary of this edition are mistaken and highly misleading). Revised transliteration, translation, and analysis [Høyrup 1993a: 246–254].

**26.** 30´ from 2 tear out, 1°30´ posit, and 5´ to ¿50 the accumulations of length and width append?.
30 *i-na* 2 ZI 1,30 GAR *ù* 5 *a-*⌈*na* ¿50 UL.GAR UŠ *ù* SAG DAḪ?]

**27.** 7 to 4, of the fourth, raise, 28 you see.
7 *a-na* 4 *re-*⟨*ba-ti*⟩ *i-ší-ma* 28 *ta-mar*

**28.** 11, the accumulations, from 28 tear out, 17 you see.
11 UL.GAR *i-na* 28 ZI 17 *ta-mar*

**29.** From 4, of the fourth, 1 tear out, 3 you see.
*i-na* 4 *re-*⟨*ba-ti*⟩ 1 ZI 3 ⌊*ta*]-*mar*

**30.** The IGI of 3 detach, 20´ you see. 20´ to 17 raise,
IGI 3 *pu-tú-*⟨*úr*⟩ 20 *ta-*⟨*mar*⟩ 20 ⌊*a-na*] 17 *i-*⟨*ší*⟩

**31.** 5°40´ you see, 5°40´, (for) the length. 20´ to 5´, the going-beyond, raise,
5,40 *ta-*⟨*mar*⟩ 5,40 [U]Š 20 *a-na* 5 *dirig i-ší*

**32.** 1´40´´ you see, 1´40´´, the appending of the length. 5°40´, (for) the length,
1,40 *ta-*⟨*mar*⟩ 1,40 *wa-ṣí-ib* UŠ 5,40 UŠ

**33.** from 11, accumulations, tear out, 5°20´ you see.
*i-na* 11 UL.GAR ZI 5,20 *ta-mar*

**34.** 1´40´´ to 5´, the going-beyond, append, 6´40´´ you see.
1,40 *a-na* 5 DIRIG DAḪ 6,40 *ta-mar*

**35.** 6´40´´, the tearing-out of the width. 5´, the step,
6,40 *n*[*a*]-*sí-iḫ* SAG 5 A.RÁ

**36.** to 5°40´, lengths, raise, 28´20´´ you see.
*a-na* 5,40 UŠ *i-ší* 28,20 *ta-mar*

**37.** 1´40´´, the appending of the length, to 28´20´´ append,
1,40 *wa-ṣí-ib* UŠ *a-na* 28,20 [DAḪ⌋

**38.** 30´ you see, 30´ the length. 5´ to 5°20´
30 *ta-mar* 30 UŠ 5 *a*-[*na* 5,20]

**39.** raise, 26´40´´ you see. 6´40´´,
*i-ši-ma* 26,40 *t*[*a-mar* 6,40]

**40.** the tearing-out of the width, from 26´40´´ you tear out,
*na-sí-iḫ* SAG *i-na* [26,40 ZI]

**41.** 20´ you see, 20´ the width.
20 *ta-mar* 20 sa[g] (...?)

Once again, the text – even this one belonging to the Susa corpus –

discusses a first-degree equation involving the length and width of a rectangle 30´×20´; this time, however, it is *solved* – which means that we are confronted with an indeterminate problem.

The first problem of the tablet has already treated the homogeneous problem

$$^{1}/_{7}(l+{}^{1}/_{4}w)\cdot 10 = l+w \ .$$

The present one, as we see, is inhomogeneous and can be translated

$$^{1}/_{7}(l+{}^{1}/_{4}w)\cdot 11 = (l+w)+5' \ .$$

It is probably significant that the addition of $^{1}/_{4}w$ is done by “appending” – as we shall see, the rectangular configuration is important this time.

Lines 19 to 23 explain the situation as done in the previous text, corresponding somehow to Figure 3 – lines 19–21 to **A**, the first half of 22 to **B**, its second half to **C**, and the first part of 23 to **D**. Then the meaning of a multiplication of $l+{}^{1}/_{4}w$ by 4 is explained – again as in TMS XVI. Lines 25–26 undertake a further transformation of the equation into one dealing with $l+w+5'$ (later spoken of as “the accumulation”) and $l$,

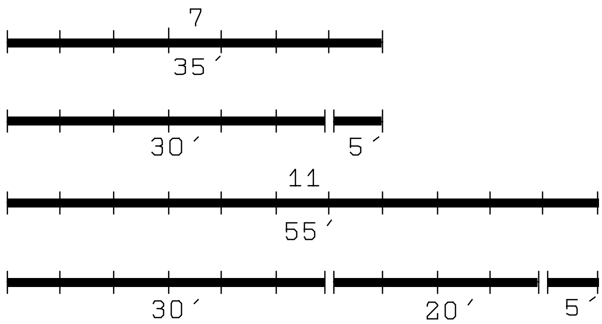


**Figure 3**.

$$^{1}/_{7}[(3l–l)–5'+(w–w)+(l+w+5')]\cdot 11 = 4\cdot(l+w+5') \ .$$

or

$$^{1}/_{7}[3l–5'+(l+w+5')]\cdot 11 = 4\cdot(l+w+5') \ ,$$

where we may notice that the result of the removal of $w$ from $w$ is regarded, literally, as *not worth speaking about*[9].

In line 27, the procedure starts for good. In symbolic translation, we first get

$$11\cdot[3(l–{}^{1}/_{3}\cdot 5')]+11\cdot(l+w+5') = 28\cdot(l+w+5')$$

[9] In the tablet VAT 7537, a similar non-numerical concept for a zero outcome of a subtraction by removal is expressed by the phrase “it is missing” (see Muroi 1991) – or, perhaps better, “it has vanished”.

and next

(*) $\quad 11\cdot(l–1'40'') = 5°40'\cdot(l+w+5')$ .

In the first problem from the tablet, a solution to the indeterminate problem

(**) $\quad 10\cdot l = 6\cdot(l+w)$

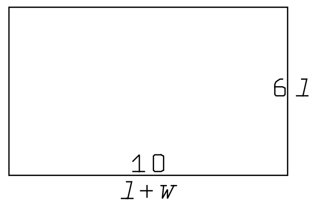


**Figure 4**.

(*viz* $l = 6$, $l+w = 10$) has been found by what appears to be an identification of both sides of the equation with the same rectangle – cf. Figure 4. The same happens in line 31 – but this time, it is $\lambda = l–1'40''$ that is identified as the "length". The entity $1'40''$ is explicitly spoken of in a kind of gerundive, as "that which shall be appended to the length [$\lambda$] [*viz* in order to get the real length $l$]" – the "appending of the length" of the translation (a Latinizing form would be "the appendendum"). The term demonstrates that $\lambda$ is really itself regarded as a length, since it is to $\lambda$, not to $l$, that $1'40''$ is to be appended. Similarly, $l+w+5'$ is thought of as the sum $\lambda+\omega$ of $\lambda$ and a modified width ($\omega$) – from which follows that $\omega = w+5'+1'40'' = w+6'40''$, where $6'40''$ must then be "that which shall be torn out from the width [$\omega$] [in order to get the real width $w$]" – the "tearing-out of the width" (lines 34–35).

A first solution to (*) is $\lambda = 5°40'$, $\lambda+\omega = 11$, whence $\omega = 11–5°40' = 5°20'$. Through multiplication by $5'$, the "step" of Figure 3 and line 20, the text finds the intended solution [$\lambda$ =] $28'20''$, [$\omega$ =] $26'40''$. Appending what should be appended to the former and tearing out what should be torn out from the latter yields $l = 30'$, $w = 20'$.

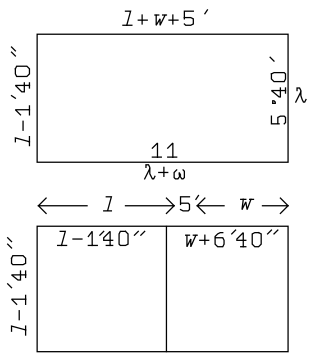


**Figure 5**.

The solution may appear unnecessarily cumbersome, but follows from the combination of two simple principles: The homogeneous equation (**) is solved with reference to Figure 4 – and the inhomogeneous equation (*) is reduced to this homogeneous equation through a "change of geometric variable", corresponding to Figure 5.

## *VAT 8391, Nº 3*[10]

### Reverse I

**3.** If from 1 BUR of surface 4 GUR of grain I have collected,
*šum-ma i-na* BÙR$^{GÁN}$ A.[ŠÀ] 4 ŠE.GUR [*am-ku-us*]

**4.** from 1 BUR of surface 3 GUR of grain I have collected,
*i-na* BÙR$^{GÁN}$ A.ŠÀ 3 ŠE.GUR *am*-[*ku-us*]

**5.** now, 2 plots. Plot over plot 10` goes beyond.
*i-na-an-na* 2 GARIM GARIM U.GÙ GARIM 10` *i-tir*

**6.** Their grain I have accumulated: 18`20.
*še-e-ši-na* GAR.GAR-*ma* 18,20

**7.** My plots what?
GARIM-*ú-a* EN.NAM

**8.** 30`, the BUR, posit. 20`, the grain which he has collected, posit
30 *bu-ra-am* GAR.RA 20 *še-am ša im-ku-sú* GAR.RA

**9.** 30`, the second BUR, posit. 15`, the grain which he has collected,
30 *bu-ra-am ša-ni-am* GAR.RA 15 *še-am ša im-ku-sú*

**9a.** posit.
GAR.RA

**10.** 10` which plot over plot goes beyond, posit.
1[0 *š*]*a* GARIM U.GÙ GARIM *i-te-ru* GAR.RA

**11.** 18`20, the accumulation of the grain, posit.
[18,20 *ku-*]*mur-ri še-im* GAR.RA

**12.** 1, projecting, posit.
[1 *wa-ṣi*]-*am* GAR.RA-*ma*

**13.** The IGI of 30`, the BUR, detach: 2´´; to the grain which he has collected
IGI 3[0 *bu-ri-im pu-tur-m*]*a* 2 *a-na še-im ša im-ku-sú*

**14.** raise, 40´, the false grain; to 10` which plot over plot goes beyond
ÍL 40 *še-um* L[UL *a-na* 1]0 *š*[*a* GARIM] U[.GÙ GARIM *i-te-r*]*u*

**15.** raise, 6`40; from 18`20, the accumulation of the grain,
*íl* 6,40 *i-na* 18,20 *ku-mur-ri še-im*

**16.** tear out: 11`40 you leave.
*ú-sú-uḫ-ma* 11,40 *te-zi-ib*

[10] Transliteration [MKT I, 321f], corrections [TMB, 110]. Translation and analysis [Høyrup 1990a: 295–299].

**17.** 11`40 which you have left, may your head retain!
11,40 *ša te-zi-bu re-eš-ka li-ki-il*

**18.** 1, projecting, to two break: 30´.
1 *wa-ṣi-am a-na ši-na ḫi-pí-ma* 30

**19.** 30´ and 30´ until twice posit:
30 *ù* 30 *a-di ši-ni-šu* GAR.RA.MA

**20.** The IGI of 30`, the BUR, detach: 2´´; to 20`, the grain which he has collected,
IGI 30 *bu-ri-im pu-ṭur-ma* 2 *a-na* 20 *še-im ša im-ku-sú*

**21.** raise, 40´; to 30´ which until twice you have posited
ÍL 40 *a-na* 30 *ša a-di ši-ni-šu ta-aš-ku-nu*

**22.** raise, 20´; may your head retain.
ÍL 20 *re-eš-ka li-ki-il*

**23.** The IGI of 30`, the second BUR, detach: 2´´.
IGI 30 *bu-ri-im ša-ni-im pu-ṭur-ma* 2

**24.** 2`` to 15`, the grain which he has collected,
2 *a-na* 15 *še-im ša im-ku-sú*

**25.** raise, 30´; to the second 30´ which you have posited, raise, 15´.
ÍL 30 *a-na* 30 *ša-ni-*[*i*]*m ša ta-aš-ku-nu* ÍL 15

**26.** 15´ and 20´, which your head retains,
15 *ù* 20 *ša re-eš-ka ú-ka-lu*

**27.** accumulate: 35´; the IGI I know not.
GAR.GAR.MA 35 *i-gi-am ú-ul i-di*

**28.** What to 35´ shall I posit
*mi-nam a-na* 35 *lu-uš-ku-un*

**29.** which 11`40 which your head retains gives me?
*ša* 11,40 *ša r*[*e-e*]*š-ka ú-ka-lu i-na-di-nam*

**30.** 20` posit. 20` to 35´ raise, 11`40 it gives you.
20 GAR.RA 20 *a-*[*na*] 35 ÍL 11,40 *it-ta-di-kum*

**31.** 20` which you have posited is the first plot;
20 *ša ta-aš-ka-*[*nu* A.]ŠÀ GARIM *iš-te-at*

**32.** from 20`, the surface of the plot, 10` which surface over surface goes beyond,
*i-na* 20 A.ŠÀ GARIM 1[0 *ša*] GARIM U.GÙ GARIM *i-t*[*e*]*-ru*

**33.** tear out, 10` the surface you leave.
*ú-sú-uḫ-ma* 10 [A.ŠÀ *te-*]*zi-ib*

(Followed by a proof, Rev. II.1–9)

Even this problem is of the first degree, but on almost all other accounts it differs from the preceding examples. It belongs on one of two tablets containing a sequence of problems dealing with the same two plots of land (**I** and **II** in the following). The rent of **I** is told to be 4 GUR of grain per BUR of land, whereas that of **II** is 3 GUR per BUR.

These units are those of practical agriculture but not those used in mathematical computations, which reduce all hollow measures to SÌLA (1 SÌLA ≈ 1 litre) and all areas to SAR (1 SAR = 1 $\text{NINDAN}^2$, cf. note 7). 1 GUR is 5` SÌLA, but the calculator does not need to multiply in order to convert the 4 and 3 GUR – they can be looked up directly in a metrological table, as 20` and 15`, respectively (lines 8 and 9), as can the value of the BUR (30` SAR). What cannot be found in a table but has to be calculated is the specific rent expressed in basic units (the "false grain", the rent that would have to be paid if the plot had been only 1 SAR).

In the present problem, we are told the difference between the areas[11] (line 5, 10` SAR) and the total rent paid for the two plots (line 6, 18`20 SÌLA). (The "now" of line 5 is probably to be read "this time is given").We start by "positing" the various data – maybe this time it simply means that we should write them down, perhaps that they are to be inserted in an adequate calculational scheme or device[12]. We also posit a number "1, projecting", to which we shall return.

Then, in lines 13–15 we find, first, the specific rent of **I** ($[30`]^{-1}\cdot 20`$ = 40´ SÌLA / SAR), and second, the rent of the part of **I** by which it exceeds **II** (6`40 SÌLA). The remaining rent (11`40 SÌLA) must then come from an area (**A**) to which **I** and **II** contribute equally.

This is where the "projecting 1" comes in. According to other texts it is the standard breadth 1 which transforms a line of length *s* [NINDAN] into a rectangle of area $1{\times}s = s$ [SAR]. In the present case, *s* is 1 NINDAN – and

[11] The Babylonian term for area is the same as the word for "field", with the only difference that the areas of mathematical problems are invariably written with the Sumerogram (A-ŠÀ), whereas real fields *may* occur in syllabic writing. In order to keep this conceptual nexus in mind, I use the translation "surface".

[12] The distinction between "positing" and keeping in the head supports the latter possibilities: if "positing" was simply "writing down", why not write down everything that was to be remembered?

the “breaking” (i.e., bisection) of the projecting 1 means that this unit area, regarded as an *average* SAR, is split into equal component parts belonging to **I** and **II** – cf. Figure 6.

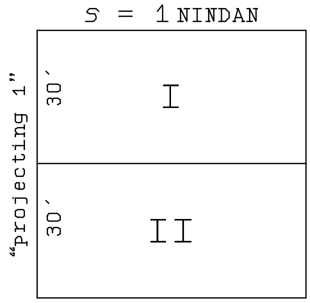


**Figure 6**.

Lines 20 to 27 calculates the rent of this average area (repeating for pedagogical reasons the computation of the specific rent of **I**) as 35´ SÌLA / SAR. **A** can thus be found as 11`40 / 35´.

35´, however is irregular, i.e., does not possess a finite sexagesimal reciprocal, and *a fortiori* no IGI listed in the table of reciprocals. The text therefore has to ask for the number which, when raised to 35´, gives 11`40 (lines 28–30). This is 20`.

At this point, a short-circuit occurs. Instead of bisecting this value, identifying one half with **II** and adding the other half to the excess in order to get **I**, it identifies the quotient directly with **I**, and finds **II** by subtracting the excess.

Such mistakes do not abound in the text material, but there are more of them (we shall meet another example in YBC 6504 N° 4). They reflect the fact that all problems were constructed backwards, and the result thus known in advance; in the present text, moreover, a copyist (who is more likely than an original author to have mixed up things) will have been familiar with the configuration from the first two problems of the tablet, and he may therefore have been tempted to “improve” a text which he had only followed imperfectly while copying.

## II. BASIC SECOND-DEGREE TECHNIQUES

### *BM 13901 Nº 1*[13]

**Obv. I**

**1.** The surface and my confrontation I have accumulated: 45´ is it. 1, the projection,

[13] Transliteration [MKT III, 1]. Translation and analysis [Høyrup 1990a: 266–270].

A.ŠÀ$^{l[am]}$ *ù mi-it-ḫar-ti ak-m[ur-m]a* 45-E 1 *wa-ṣi-tam*

**2.** you posit. The moiety of 1 you break, 30´ and 30´ you make hold each other.
*ta-ša-ka-an ba-ma-at* 1 *te-ḫe-pe* [3]0 *ù* 30 *tu-uš-ta-kal*

**3.** 15´ to 45´ you append: 1 makes 1 equilateral. 30´ which you have made hold
15 *a-na* 45 *tu-ṣa-ab-ma* 1-⌞E] 1 ÍB.SI$_8$ 30 *ša tu-uš-ta-ki-lu*

**4.** in the inside of 1 you tear out: 30´ the confrontation.
*lìb-ba* 1 *ta-na-sà-aḫ-ma* 30 *mi-it-ḫar-tum*

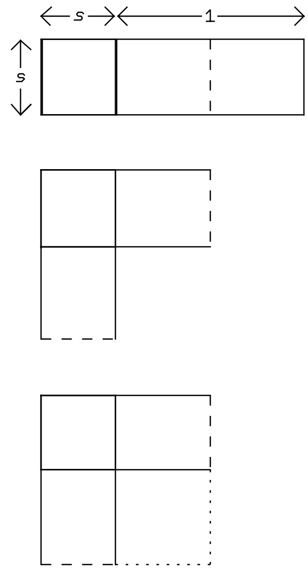


**Figure 7**.

This is, in modern terms, a normalized second-degree problem with one unknown. It deals with a square, the Babylonian concept of which requires some explanation. To us, *a square* is a "figure", i.e., an area contained by a border (in agreement with *Elements* I, definitions 14 and 22); it *is* its area (say, 9 m$^{2)}$ and *has* a side (3 m). To the Babylonians, instead, the frame was the essential aspect of the configuration – the name of the square, the *mitḫartum*, translated "confrontation", is a verbal noun referring to a situation characterized by the confrontation of equals. To them, the square *is* its side (the "confrontation" of 30´ NINDAN) and *has* an area (15´ SAR) – corresponding to that other Greek concept of a square, the much-discussed *dýnamis*, cf. [Høyrup 1990b].

If *s* designates the side, the problem can thus be translated into

$$s^2+s = 45' ,$$

and the numerical steps of the solution correspond exactly to those by which we would solve this equation. The "projection", "breaking" and "moiety" show, however, that the Babylonian calculator worked within a different – geometrical – framework – see Figure 7. The statement "accumulates" the area and the side, i.e., adds their measuring numbers. In order to make this concretely meaningful we conceptualize the side *s* as provided with a "projection 1"; we then know that the total area of the square □(*s*) together with the adjacent rectangle ⊏⊐(1,*s*) is 45´[14]. Next we

[14] For convenience I shall henceforth use the symbol □(*s*) for the geometric square on *s*, and ⊏⊐(*a*,*b*) for the rectangle contained by *a* and *b*.

“break” the projection. “Breaking” is a process which bisects into “natural” or “necessary” halves (“moieties” in the translation), halves which could not be (say) 29´ and 31´ times the entity in question. It finds the radius of the circle from the diameter, the average between opposite sides when we have to calculate the area of a trapezium, etc. Even in cases where only one of the moieties is used, “breaking” is distinguished sharply from taking a merely incidental half through multiplication by 30´ (thus for instance in AO 8862 N° 2, see below, p. 31).

In the present case, the halves can indeed be nothing but halves, since the outer half of the rectangle has to be moved so as to “hold” a square together with the part that remains in place[15]. This produces a gnomon of area 45´, which is completed by the square ☐(30´) = 15´. The area of the completed square is thus 45´+15´ = 1, and it is told that “1 makes 1 equilateral” – i.e., when (the first) 1 is laid out as a square, (the second) 1 will be the (equilateral) side. “Tearing out” that part of the rectangle which was “made hold”, i.e., which was moved, leaves the side of the square, as 1–30´ = 30´.

No attempt is made in the text to prove explicitly that the outcome of this geometrical cut-and-paste procedure is identical with the side – but that it really is can be “seen” immediately. The procedure is thus not blind, not the outcome of a trial-and-error play with numbers as sometimes assumed; but we may label it “naive”, in contrast to the “critical” style of Euclid’s *Elements*, where the explicit concern for proof is paramount. In this respect the Babylonian technique is akin to modern school algebra (at least as it looked before the new math movement): even here, the correctness of operations is mostly obvious but not subjected to explicit proof.

[15] The literal meaning of the term (*šutākulum* / *šutakūlum*) has been subject to much discussion. It is a reciprocal causative, either of *akālum*, “to eat”, or from *kullum*, “to hold”. Mostly the former derivation has been accepted, because of a Sumerographic writing by means of KÚ, “to eat”. Often, however, the relative clause of line 3, “which you have made hold / eat”, is replaced by a verbal noun that cannot derive from *akālum* (cf. p. 21, YBC 6967, rev. 1, where it is occurs as “the made-hold”), which excludes the habitual interpretation; the Sumerographic writing, on the other hand, is easily explained as a pun-like transfer, of which there are many in the cuneiform script.

***BM 13901 Nº 2*[16]**

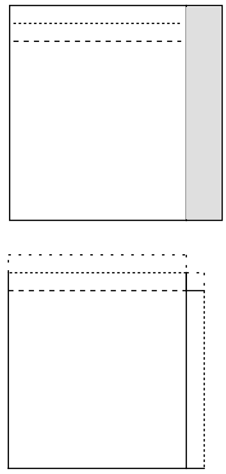

**Figure 8.**

**5.** My confrontation inside the surface I have torn out: 14`30 is it. 1, the projection,
*mi-it-ḫar-ti lìb-bi* A.ŠÀ [*a*]*s-sú-uḫ-ma* 14,30-E 1 *wa-si-tam*

**6.** you posit. The moiety of 1 you break, 30´ and 30´ you make hold each other,
*ta-ša-ka-an ba-ma-at* 1 *te-ḫe-pe* 30 *ù* 30 *tu-uš-ta-kal*

**7.** 15´ to 14`30 you append: 14`30°15´ makes 29°30´ equilateral.
15 *a-*⌈*na* 14,30 *tu-sa-*]*ab-ma* 14,30,15-E 29,30 ÍB.SI$_8$

**8.** 30´ which you have made hold to 29°30´ you append: 30 the confrontation.
30 *ša tu-uš-ta-ki-lu a-na* 29,30 *tu-ṣa-ab-ma* 30 *mi-it-ḫar-tum*

This problem follows directly after the previous one in a tablet containing in total 24 problems about squares. The question is equally simple:

$$\square(s)–s = 14`30 ,$$

and it is dealt with in a similar way: Removal of a side (provided once again with a "projection"; shaded in Figure 8) leaves us with a rectangle whose length exceeds its width by a known amount (*viz* 1, the "projection"), and whose area is known to be 14`30. This excess is broken, the outer half moved so as to make the two parts "hold" a completing square $\square(^1/_2)$ = 15´. This is appended to the gnomon, which gives us an area of the completed square equal to 14`30°15´ and a corresponding "equilateral" 29°30´. Putting back in place that half of the excess which was moved in order to "hold" restores the side of the original square.

[16] Transliteration [MKT III, 1], translation and analysis [Høyrup 1990a: 270f].

## *BM 13901 Nº 14*[17]

**Obv. II**

**44.** The surfaces of my two confrontations I have accumulated: 25´25´´.
A.ŠÀ *ši-ta mi-it-ḫa-ra-ti-ia ak-mur-ma* ⸢25,]25

**45.** The confrontation, two-third of the confrontation and 5´ NINDAN.
*mi-it-ḫar-tum ši-ni-pa-at mi-it-ḫar-tim* [*ù* 5 NIND]AN

**46.** 1 and 40´ and 5´ over-going 40´ you inscribe.
1 *ù* 40 *ù* 5 [*e-le-nu* 4]0 *ta-la-pa-at*

**47.** 5´ and 5´ you make hold each other, 25´´ inside 25´25´´ you tear out:
5 *ù* 5 [*tu-uš-ta-kal 25 lìb-bi 25,25 ta-na-sà-aḫ-ma*]

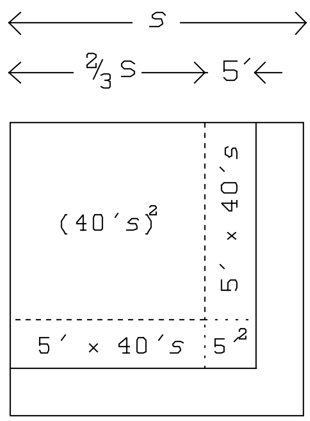


**Figure 9**.

**Rev. I**

**1.** 25´ you inscribe. 1 and 1 you make hold each other, 1. 40´ and 40´ you make hold each other,
[25 *ta-la-pa-at* 1 *ù* 1 *tu-uš-ta-kal* 1 40 *ù* 40 *tu-uš-ta-kal*]

**2.** 26´40´´ to 1 you append: 1°26´40´´ to 25´ you raise:
[26,40 *a-na* 1 *tu-ṣa-ab-ma* 1,26,40 *a-na* 25 *ta-na-ši-ma*]

**3.** 36´6´´40´´´ you inscribe. 5´ to 40´ you raise: 3´20´´
[36,6,40 *ta-la-pa-at* 5 *a-na* 4]0 *t*[*a-na-ši-ma* 3,20]

**4.** and 3´20´´ you make hold each other, 11´´6´´´40´´´´ to 36´6´´40´´´ you append:
[*ù* 3,20 *tu-uš-ta-kal* 11,6,40] *a-na* 3[6,]6,40 [*tu-sa-ab-ma*]

**5.** 36´17´´46´´´40´´´´ makes 46´40´´ equilateral. 3´20´´ which you have made hold
[36,17,46,40-E 46,40 ÍB.SI$_8$ 3,]20 *ša tu-uš-ta-ki*[*-lu*]

**6.** inside 46´40´´ you tear out: 43´20´´ you inscribe
[*lìb-bi* 46,40 *ta-na-sà-aḫ-*]*ma* 43,20 *ta-la-pa-a*[*t*]

**7.** The IGI of 1°26´40´´ is not detached. What to 1°26´40´´
[IGI 1,26,40 *ú-la ip-pa-ṭ*]*a-ar mi-nam a-na* 1,2[6,4]0

---

[17] Transliteration [MKT III, 3]; translation and analysis [Høyrup 1990a: 306–309]. The text of the problem is rather damaged; due to the parallels in Nº 24, however, all restitutions apart perhaps from minute details seem certain.

**8.** shall I posit which 43´20´´ gives me? 30´ its *bandûm*.
[*lu-uš-ku-un ša* 43,20 *i-n*]*a-di-nam* 30 *ba-an-da-šu*

**9.** 30´ to 1 you raise: 30´ the first confrontation.
[30 a-na 1 ta-na-ši-ma 30] mi-it-ḫar-tum iš-ti-a-at

**10.** 30´ to 40´ you raise: 20´, and 5´ you append:
[30 *a-na* 40 *ta-na-ši-ma* 20] *ù* 5 *tu-sa-ab-ma*

**11.** 25´ the second confrontation
[25 *mi-it-ḫar-t*]*um ša-ni-tum*

After a number of intermediate steps where the single techniques are introduced and trained, the same "square" text comes to this problem about two squares:

$$\square(s_1)+\square(s_2) = 25´25´´ \ , \quad s_2 = {}^2\!/_3 s_1+5´$$

Once again, the numerical steps run parallel to what we would do (if submitted to the constraints of the sexagesimal system). In order to express $\square(s_1)$ and $\square(s_2)$ in terms of a third square $\square(s)$ and its side $s$, it "inscribes" 1 ($s_1 = 1\cdot s$), ${}^2\!/_3$ = 40´ and 5´ ($s_2 = 40´\cdot s+5´$) in line 46. As seen in Figure 9, $\square(s_2)$ decomposes into $\square(40´s)$, $2\sqsubset\!\sqsupset(5´,40´s)$ and $\square(5’)$, which the text identifies without difficulty with $\square(40´)$ (treated as the number $40´^2 = 26´40´´$) times $\square(s)$, 2 times $\sqsubset\!\sqsupset(40´\cdot5´,s)$[18], and 25´´. $\square(s_1)$, of course, is $\square(1)$ = 1 times $\square(s)$.

The problem is thus reduced to

$$1°26´40\square(s)+2\cdot\sqsubset\!\sqsupset(3´20´´,s) = 25´25´´–25´´ = 25´ \ .$$

This problem cannot be normalized in the way we would normally do it, since 1°26´40´´ does not divide 25´. That is at least one reason why the text chooses a different path. In general, the Babylonians would solve problems

$$\alpha\square(s)+\beta s = q$$

and

$$\sqsubset\!\sqsupset(\alpha s,s)+\beta s = q$$

by a change of variable, reducing them to

[18] That this, and not $2\cdot\sqsubset\!\sqsupset(5´\cdot40´)$ times $\sqsubset\!\sqsupset(1,s)$ is meant follows from the use of "raising" instead of "making hold".

$$\square(\alpha s)+\beta(\alpha s) = \alpha\cdot q\ .$$

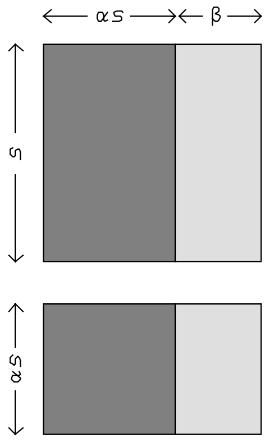

**Figure 10**.

Geometrically, this corresponds to a change of scale in one direction, by which the rectangle $\sqsubset\!\sqsupset(\alpha s,s)$ is transformed into a square (this trick we shall encounter time and again in the following) – see Figure 10. We observe that the scaling transforms the $\beta$ sides of $\square(s)$ into $\beta$ sides of $\square(\alpha s)$.

$\alpha Q$ is found in (Rev. I) line 2. Since $\beta$ is already known to be twice 3´20´´, no bisection is needed in order to produce the sides of the completing square, but apart from that everything runs as in problem N° 1 until line 6, where $\alpha s$ is found to be 43´20´´. $s$ itself is found to be 30´ through division by the irregular number $\alpha$ = 1°26´40´´ (the term *bandûm*, apparently a Sumerian loanword, evidently designates “what shall be put alongside the divisor”, which may indeed be the Sumerian etymology). Line 9 shows that the text really operates in terms of a new $s$ and not in terms of $s_1$, since $s_1$ is found as $1\cdot s$. Lines 10-11 finally find $s_2$.

Beyond the secure hand demonstrated by the text, its most important feature is how it dispenses with the doubling of 3´20´´ and the ensuing bisection. This illustrates that the Babylonians calculators did not operate with fixed standard algorithms (as claimed in much of the secondary literature). Theirs was a flexible understanding, allowing them to make shortcuts when these were allowed by a particular situation.

### *YBC 6967*[19]

**Obverse**

**1.** The *igibûm* over the *igûm* 7 goes beyond
[IGI.B]I *e-li* IGI 7 *i-ter*

**2.** *igûm* and *igibûm* what?
[IGI] *ù* IGI.BI *mi-nu-um*

**3.** You, 7 which the *igibûm*
*a*[*t-t*]*a* 7 *ša* IGI.BI

19 Transliteration [MCT, 129]. Translation and analysis [Høyrup 1990a: 262–266].

**4.** over the *igûm* goes beyond
UGU IGI *i-te-ru*

**5.** to two break: 3°30´;
*a-na ši-na ḫi-pí-ma* 3,30

**6.** 3°30´ together with 3°30´
3,30 *it-ti* 3,30

**7.** make hold each other: 12°15´.
*šu-ta-ki-il-ma* 12,15

**8.** To 12°15´ which comes up for you
*a-na* 12,15 *ša i-li〈-a〉-kum*

**9.** 1` the surface append: 1`12°15´.
[1 A.ŠA$^{l}$]$^{a\text{-}am}$ *sí-ib-ma* 1,12,15

**10.** The equilateral of 1`12°15´ what? 8°30´.
[ÍB.SI$_8$ 1],12,15 *mi-nu-um* 8,30

**11.** 8°30´ and 8°30´, its counterpart, lay down.
[8,30 *ù*] 8,30 *me-ḫe-er-šu i-di-ma*

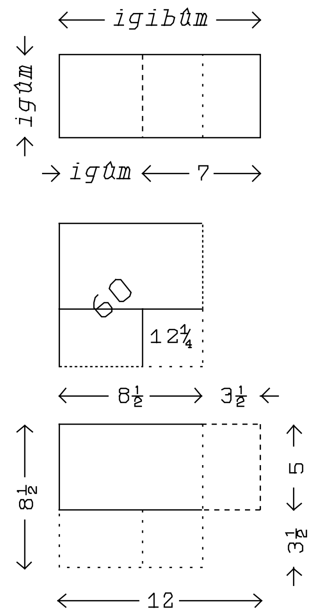


**Figure 11**.

**Reverse**

**1.** 3°30´, the made-hold,
3,30 *ta-ki-il-tam*

**2.** from one tear out,
*i-na iš-te-en ù-su-uḫ*

**3.** to the other append.
*a-na iš-te-en sí-ib*

**4.** The first is 12, the second is 5.
*iš-te-en 12 ša-nu-um* 5

**5.** 12 is the *igibûm*, 5 is the *igûm*
12 IGI.BI 5 *i-gu-um*

More common than "square problems" in the Babylonian corpus are "rectangle problems". Very often, complex problems reduce to the simpler cases of finding the sides of a rectangle of which the area and either the sum of the two sides or their difference is known.

The present problem is itself of the latter type, apart from the fact that it does not deal with geometry but with two numbers belonging together in the table of reciprocals – i.e., two numbers whose product is 1 or (in the actual case) 60. They are spoken of as *igûm* and *igibûm*, Akkadian pronunciations of IGI and IGI.BI, “the reciprocal and its reciprocal”.

If the latter is $x$ and the former $y$ (these symbols, which we habitually see as unknown *numbers*, are adequate in the present case), the problem is

$$x\text{–}y = 7\ , \quad x\cdot y = 60\ .$$

Whereas *we* are accustomed to represent (e.g.) the length and width of a rectangle by unknown numbers, the Babylonian calculator represents his unknown numbers by his standard instruments – measurable line segments – and speaks in line 9 of the product as a “surface”. The procedure is already familiar (see Figure 11, and compare with Figure 7 and Figure 8): We know that the length (*igûm* = $x$) of the rectangle exceeds its width (*igibûm* = $y$) by 7. This excess is bisected and the rectangle transformed into a gnomon, still of area 60, which is appended to the completing square $\square(\frac{7}{2}) = 12\frac{1}{4}$[20]. The side of the completed square – its “equilateral” – must then be $8\frac{1}{2}$, which is “laid down” together with “its counterpart” (etymologically related to the “confrontation”), as two sides of the completed square. “Tearing out” that part (“the made-hold”) of the excess which was moved in order to “hold” the complement we get the *igûm;* putting it back to its original position we restore the *igibûm*.

The text illustrates that the Babylonian operation with lines and areas was really *an algebra*, if this is understood as analytic procedures in which unknown quantities are *represented* by functionally abstract entities – numbers in our algebra, measurable line segments and areas in the Babylonian technique.

### *TMS IX, Parts A and B*[21]

**Part A**

**1.** The surface and 1 length accumulated, 40´. ¿30, the length,? 20´ the width.
A.ŠÀ *ù* 1 UŠ UL.GAR 4[0 ?30 UŠ? 20 SAG]

**2.** As 1 length to 10´ the surface, has been appended,

20 Normally, it is the completing square that is appended; but since both addends are already in place, one order is just as good as the other.

21 Transliteration [TMS, 63], corrections, translation and analysis [Høyrup 1990a: 320–327].

*i-nu-ma* 1 UŠ *a-na* 10 ⌈A.ŠÀ DAḪ]

**3.** or 1 (as) base to 20´, the width, has been appended,
*ú-ul* 1 KI.GUB.GUB *a-na* 20 [SAG DAḪ]

**4.** or 1°20´ ¿is posited? to the width which together with the length ¿holds? 40´
*ú-ul* 1,20 *a-na* SAG *šà* 40 *it*-⌈*ti* UŠ ¿NIGIN GAR?]

**5.** or 1°20´ together with 30´ the length holds, 40´ is its name.
*ú-ul* 1,20 *it*-⟨*ti*⟩ 30 UŠ NIG[IN] 40 *šum*-[*šu*]

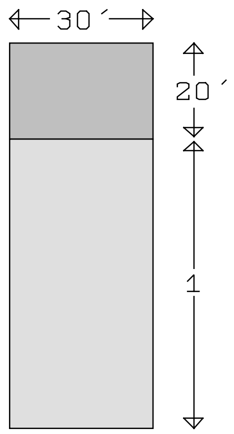


**Figure 12.**

**6.** Since so, to 20´ the width, which is said to you,
*aš-šum ki-a-am a-na* 20 SAG *šà qa-bu-ku*

**7.** 1 append: 1°20´ you see. Out from here
1 DAḪ-*ma* 1,20 *ta-mar iš-tu an-ni-ki-a-am*

**8.** you ask. 40´ the surface, 1°20´ the width, the length what?
*ta-šà-al* 40 A.ŠÀ 1,20 SAG UŠ *mi-nu*

**9.** 30´ the length. So the having-been-made.
[30 UŠ *k*]*i-a-am ne-pé-šum*

**Part B**

**10.** Surface, length and width accumulated, 1. By the Akkadian (method).
[A.ŠÀ UŠ *ù* SAG U]L.GAR 1 *i-na ak-ka-di-i*

**11.** 1 to the length append. 1 to the width append. Since 1 to the length is appended,
[1 *a-na* UŠ DAḪ] 1 *a-na* SAG DAḪ *aš-šum* 1 *a-na* UŠ DAḪ

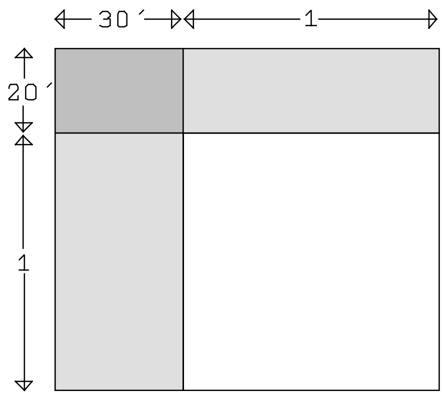


**Figure 13.**

**12.** 1 to the width is appended, 1 and 1 make hold, 1 you see.
[1 *a-na* SAG D]AḪ 1 *ù* 1 NIGIN 1 *ta-mar*

**13.** 1 to the accumulation of length, width and surface append, 2 you see.
[1 *a-na* UL.GAR UŠ] SAG *ù* A.ŠÀ DAḪ 2 *ta-mar*

**14.** To 20´ the width, 1 append, 1°20´. To 30´ the length, 1 append, 1°30´.
[*a-na* 20 SAG 1 DA]Ḫ 1,20 *a-na* 30 UŠ 1 DAḪ 1,30

**15.** ¿Since? a surface, that of 1°20´ the width, that of 1°30´ the length,

[¿*aš-šum*? A.Š]À *šà* 1,20 SAG *šà* 1,30 UŠ

**16.** ¿the length together with? the width, are made hold, what is its name?
[¿UŠ *it-ti*? SA]G *šu-ta-ku-lu mi-nu šum-šu*

**17.** 2 the surface.
2 A.ŠÀ

**18.** So the Akkadian (method).
*ki-a-am ak-ka-du-ú*

This text (once again from Susa) belongs to the same didactically explicit genre as the first-degree texts TMS XVI and TMS VII, both discussed above. This one, however, explains some of the basic second-degree techniques in Parts A and B, before applying these to a complex problem in Part C.

All three parts deal with the same rectangle ⊏⊐(30´,20´). The tablet is damaged, but Part A clearly presupposes in its explanation that these dimensions are known, as is the area (10´). It discusses what to do when the sum of area and length is known, ⊏⊐($l$,$w$)+$l$ = 40´. It is immediately taken for granted ("since ...") that this means that the width is prolonged by 1 (cf. Figure 12). Then follow a sequence of reformulations ("or ... or ... or ...", much in the vein of modern mathematical parlance). All in all, the total area 40´ is seen to be the rectangle held by the length and the width prolonged by the "base" 1. In the end it is told how, if the width 20´ and the total area 40´ are known, the length can be found to be 30´.

Part B still presupposes the known values of $l$ and $w$ in its explanations but now treats the situation where ⊏⊐($l$,$w$)+$l$+$w$ = 1, and tells us how to apply "the Akkadian method". As shown in Figure 13, this implies that both length and width are prolonged by 1, and hence also that a completing square □(1,1) be appended to the area. This "surface 2" then has the length 1°30´ and the width 1°20´.

Since the feature which distinguishes this part most clearly from the preceding one is the quadratic completion, we may safely assume that this trick is what carried the name "the Akkadian [method]".

## III. COMPLEX SECOND-DEGREE PROBLEMS

### *TMS IX, Part C*[22]

**19.** Surface, length and width accumulated, 1 the surface. 3 lengths, 4 widths accumulated,
A.ŠÀ UŠ *ù* SAG UL.GAR 1 A.ŠÀ 3 UŠ 4 SAG UL.GAR

**20.** its 17th to the width appended, 30´.
[17]-*ti-šu a-na* SAG DAḪ 30

**21.** You, 30´ to 17 go: 8°30´ you see.
[ZA.]E 30 *a-na* 17 *a-li-ik-ma* 8,30 [*t*]*a-mar*

**22.** To 17 widths 4 widths append, 21 you see.
[*a-na* 17 SAG] 4 SAG DAḪ-*ma* 21 *ta-mar*

**23.** 21 as much as of widths posit. 3, of three of lengths,
[21 *ki-*]*ma* SAG GAR 3 *šà-la-aš-ti* UŠ

**24.** 3, as much as lengths posit. 8°30´, what is its name?
[3 *ki*]-*ma* UŠ GAR 8,30 *mi-nu šum-šu*

**25.** 3 lengths and 21 widths accumulated.
[3] UŠ *ù* 2[1 SA]G UL.GAR

**26.** 8°30´ you see
8,30 *ta-mar*

**27.** 3 lengths and 21 widths accumulated.
[3] UŠ *ù* 21 SAG UL.[GAR]

**28.** Since 1 to the length is appended and 1 to the width is appended, make hold:
[*aš-šum* 1 *a-na*] UŠ DAḪ [*ù* 1 *a*]-*na* SAG DAḪ NIGIN-*ma*

**29.** 1 to the accumulation of surface, length and width append, 2 you see,
1 *a-na* UL.GAR A.ŠÀ UŠ *ù* SAG DAḪ 2 *ta-mar*

**30.** 2 the surface. Since the length and the width of 2 the surface,
[2 A.]ŠÀ *aš-šum* UŠ *ù* SAG *šà* 2 A.ŠÀ

**31.** 1°30´, the length, together with 1°20´, the width, are made hold,
[1,30 UŠ *it*]-*ti* 1,20 SAG *šu-ta-ku-lu*

**32.** 1 the appended of the length and 1 the appended of the width,
[1 *wu-ṣú-*]*bi* UŠ *ù* 1 *wu-ṣú-bi* SAG

**33.** make hold ¿1 you see? 1 and 1, the various things, accumulate, 2 you see.

---

[22] Transliteration [TMS, 63f], corrections, translation and analysis [Høyrup 1990a: 321–325, 327f].

[NIGIN ¿1 *ta-mar*? 1 *ù* 1 ¿...?] ḪI.A UL.GAR 2 *ta-mar*

**34.** 3 (¿...?), 21 (¿...?) and 8°30´ accumulate, 32°30´ you see;
[3 ¿...? 21 ¿...? *ù* 8,30 UL.GAR] 32,30 *ta-mar*

**35.** so you ask.
[*ki-a*]-*am ta-šà-al*

**36.** ... of widths, to 21, accumulate/-ion:
[...].TI SAG *a-na* 21 UL.GAR-*ma*

**37.** ... to 3, lengths, raise,
[...] *a-na* 3 UŠ *i-ší*

**38.** 1`3 you see. 1`3 to 2, the surface, raise:
[1,3 *ta-mar* 1,3 *a*]-*na* 2 A.ŠÀ *i-ši-ma*

**39.** 2`6 you see, ¿2`6 the surface?. 32°30´ the accumulation break, 16°15´ you see.
[2,6 *ta-mar* ¿2,6 A.ŠÀ?] 32,30 UL.GAR *ḫi-pí* 16,15 *ta-*⟨*mar*⟩

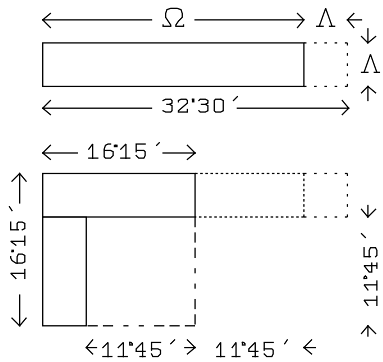


**Figure 14**.

**40.** 16°15´ the counterpart posit, make hold,
{16,15 *ta-mar*} 16,15 GABA GAR NIGIN

**41.** 4`24°3´45´´ you see. 2`6
4,[24,]3,45 *ta-mar* 2,6 [¿erasure?]

**42.** from 4`24°3´45´´ tear out, 2`18°3´45´´ you see.
*i-na* 4,[2]4,3,45 ZI 2,18,3,45 *ta-mar*

**43.** What is made equilateral? 11°45´ is made equilateral, 11°45´ to 16°15´ append,
*mi-na* ÍB.SI 11,45 ÍB.SI 11,45 *a-na* 16,15 DAḪ

**44.** 28 you see. From the 2nd tear out, 4°30´ you see.
28 *ta-mar i-na* 2-KAM ZI 4,30 *ta-mar*

**45.** The IGI of 3, the lengths, detach, 20´ you see. 20´ to 4°30´
IGI 3-*ti* UŠ *pu-ṭú́r* 20 *ta-mar* 20 *a-na* 4,[30]

**46.** raise: 1°30´ you see,
{20 *a-na* 4,30} *i-ši-ma* 1,30 *ta-mar*

**47.** 1°30´ the length of 2 the surface. What to 21, the widths, shall I posit
1,30 UŠ *šà* 2 A.Š[À *mi-na*] *a-na* 21 SAG [*lu-uš-ku-un*]

**48.** which 28 gives me? 1°20´ posit, 1°20´ the width
*šà* 28 *i-na-di*[-*na* 1,20 G]AR 1,20 SAG

**49.** of 2 the surface. Turn back. 1 from 1°30´ tear out,
*šà* 2 A.ŠÀ *tu-úr* 1 *i-na* 1,[30 ZI]

**50.** 30´ you see. 1 from 1°20´ tear out,
30 *ta-mar* 1 *i-na* 1,20 Z[I]

**51.** 20´ you see.
20 *ta-mar*

Part C of the same didactical tablet combines the equation of Part B with an abstruse linear condition:

$$\sqsubset\!\sqsupset(l,w)+l+w = 1\ , \quad {}^{1}/_{17}(3l+4w)+w = 30´\ .$$

At first it transforms the latter equation by means of the techniques taught in TMS XVI: multiplying by 17, finding the total coefficients of $l$ and $w$. As summed up in line 26f,

$$3l+21w = 8°30´\ .$$

Next it repeats the trick of Part B, showing that a "surface 2", with length 1°30´ and width 1°20´, presupposes that 1 is appended to both length and width – putting $\lambda = l+1$, $\omega = w+1$, we get $\sqsubset\!\sqsupset(\lambda,\omega) = 2$. Moreover (the damages to lines 33 and 34 prevents us from knowing the exact formulation), $3\lambda+21\omega = 32°30$. Finally, if $\Lambda = 3\lambda$, $\Omega = 21\omega$ ($\Lambda$ and $\Omega$, in contrast to $\lambda$ "the length of 2 the area" and $\omega$ "the width of 2 the area", carry no name of their own in the text, whereas their sum is spoken of in line 39 as "the accumulation")

$$\Lambda+\Omega = 32°30\ , \quad \sqsubset\!\sqsupset(\Lambda,\Omega) = (3\cdot21)\cdot2 = 2`6\ .$$

This standard form of the problem is obtained in line 39, after which a normal cut-and-paste procedure starts (cf. Figure 14): the sum of length and width is bisected and the counterpart of the moiety posited so as to hold a completed square, whose area must be $\square(16°15´) = 4`24°3´45´´$. From this is torn out the area of the rectangle, transformed into a gnomon, leaving for the completing square an area 2`18°3´45´´. This makes 11°45´ "equilateral", which is appended to the first side of the completed square (horizontal in Figure 14) and torn out from its vertical counterpart. This order differs from the one of YBC 6967 (and, in general, from the one which we find when the difference between length and width is known). The reason is straightforward. In YBC 6967, what we tear out and append is *the same* entity, that half of the difference which was "made hold";

evidently, this cannot be appended before it has been made available by being torn out. In the present case, what we append and tear out are the sides of the newly produced completing square, of which we can dispose freely; in this situation, the Babylonians obey the same psychological "law" that make us prefer expressions $a{\pm}b$ to the alternative $a{\mp}b$ when the choice is free, and which made the author of BM 13901 treat the question $\square(s)+s$ = 45´ before the question $\square(s)-s$ = 14`30.

Appending and tearing out gives the values of Λ (= 4°30´) and Ω (= 28), from which λ = 1°30´ and ω = 1°20´ follow. Finally we "turn back" to the original rectangle by tearing out 1 from each.

### *AO 8862 Nº 2*[23]

**I**

**30.** Length, width. Length and width
UŠ SAG UŠ *ù* SAG

**31.** I have made hold each other. A surface I have built.
*uš-ta-ki-il$_5$-ma* A.ŠÀ$^{lam}$ *ab-ni*

**32.** I went around (it). The half of the length
*a-sà-ḫi-ir mi-ši-il$_5$* UŠ

**33.** and the third of the width
*ù ša-lu-uš-ti* SAG

**34.** to the inside of my surface
*a-na li-bi* A.ŠÀ-*ia*

**35.** I have appended: 15.
[*ú*-]*si-ib-ma* 15

**36.** I turned back. Length and width
[*a-t*]*u-úr* UŠ *ù* SAG

**37.** I have accumulated: 7.
[*ak*-]*mu-ur-ma* 7

**II**

**1.** Length and width what?
UŠ *ù* SAG *mi-nu-um*

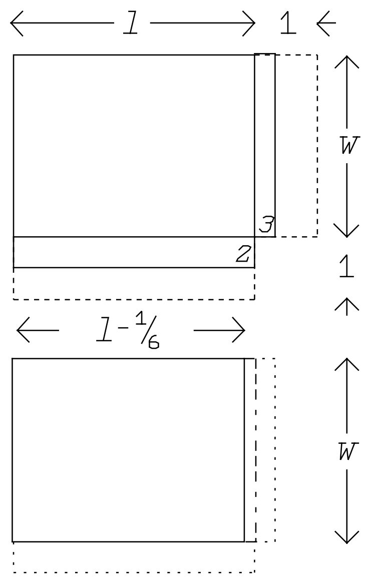


**Figure 15.**

[23] Transliteration [MKT I, 109f]. Translation and analysis [Høyrup 1990a: 311–317].

**2.** You, by your making,
*at-ta i-na e-pe-ši-i-ka*

**3.** 2 (as) inscription of the half
[2 *n*]*a-al-p*[*a*]*-at-ti mi-iš-li-im*

**4.** and 3 (as) inscription
[*ù*] 3 *na-al-pa-ti*

**5.** of the third you inscribe:
[*ša-*]*lu-uš-ti ta-l*[*a*]*-pa-at-ma*

**6.** The IGI of 2, 30´, you detach:
IGI 2-BI 30 *ta-pa-ṭar-ma*

**7.** 30´ steps of 7, 3°30´; to 7,
30 A.RÁ 7 3,30 *a-na* 7

**8.** the things accumulated, length and width,
*ki-im-ra-tim* UŠ *ù* SAG

**9.** I bring:
*ub-ba-al-ma*

**10.** 3°30´ from 15, my things accumulated,
3,30 *i-na* 15 *ki-i*[*m*]*-ra-ti-i-a*

**11.** cut off:
*ḫu-ru-us*$_4$*-ma*

**12.** 11°30´ the remainder.
11,30 *ša-pi-il*$_5$*-tum*

**13.** Go not beyond. 2 and 3 make hold each other:
*l*[*a*] *wa-t*[*ar*] 2 *ù* 3 *uš-ta-kal-ma*

**14.** 3 steps of 2, 6.
3 A.RÁ 2 6

**15.** The IGI of 6, 10´ it gives you.
IGI 6 GÁL 10 *i-na-di-kum*

**16.** 10´ from 7, your things accumulated,
10 *i-na* 7 *ki-im-ra-ti-i-ka*

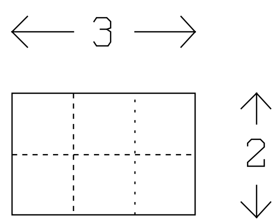


**Figure 16**.

**17.** length and width, I tear out:
UŠ *ù* SAG *a-na-sà-aḫ-ma*

**18.** 6°50´ the remainder.
6,50 *ša-pi-il*$_5$*-tum*

**19.** Its moiety, that of 6°50´, I break:
BA.A-*š*[*u*] *ša 6,50 e-ḫe-pe-e-ma*

**20.** 3°25´ it gives you.
3,25 *i-na-di-ku*

**21.** 3°25´ until twice
3,25 *a-di -ši-ni-šu*

**22.** you inscribe; 3°25´ steps of 3°25´,
*ta-la-pa-at-ma* 3,25 A.RÁ 3,25

**23.** 11°40´25´´; from the inside
11,40,[25] *i-na li-bi*

**24.** 11°30´ I tear out:
11,30 *a-na-sà-aḫ-ma*

**25.** 10´25´´ the remainder. ⟨10´25´´ makes 25´ equilateral⟩.
10,25 *ša-pi-il$_5$-tum* ⟨10,25-E 25 ÍB.SI$_8$⟩

**26.** To the first 3°25´
*a-na* 3,25 *iš-te-en*

**27.** 25´ you append: 3°50´,
25 *tu-ṣa-am-ma* 3,50

**28.** and (that) which from the things accumulated of
*ù ša i-na ki-im-ra-at*

**29.** length and width I have torn out
UŠ *ù* SAG *a*[*s*]*-sà-aḫ-ma*

**30.** to 3°50´ you append:
*a-na* 3,50 *tu-ṣa-am-ma*

**31.** 4 the length. From the second 3°25´
4 UŠ *i-na* 3,25 *ša-ni-im*

**32.** 25´ I tear out: 3 the width.
25 *a-na-sà-aḫ-ma* 3 SAG

**32a.** 7 the things accumulated.
7 *ki-im-ra-tu-ú*

**32b.** 4, the length
3, the width
12, the surface

4 UŠ
3 SAG
12 A.ŠÀ

The problem deals with a field which is determined by *one* length and *one* width: that is, within the universe of Babylonian mathematics, a rectangular field. Its format is close to a surveyors' riddle – "I have laid out a field, I have gone around it, ...". The underlying problem, however,

$$\sqsubset\!\sqsupset(l,w)+{}^1\!/\!_2\,l+{}^1\!/\!_3\,w = 15\ , \quad l+w = 7\ ,$$

is close to the preceding text, and the solution *might* have followed the same pattern:

$$\sqsubset\!\sqsupset(\lambda,\omega) = 15+\sqsubset\!\sqsupset({}^1\!/\!_2,{}^1\!/\!_3) = 15°10'\ , \quad \lambda+\omega = 7+{}^1\!/\!_3+{}^1\!/\!_2 = 7°50'\ ,$$

$\lambda = l+{}^1\!/\!_3$, $\omega = w+{}^1\!/\!_2$. This, however, is not what happens. The length and the width are imagined with the standard breadth 1 in their natural location, which explains why something can be "brought" to them in lines 8–9, and which fits the use of the plural ("the things accumulated") when their sum is spoken of – indeed, they remain separate.

2 and 3 are "inscribed" as denotations of the half and the third (something like the italicized numbers of Figure 15 is a possible interpretation). In the next step, half of the accumulation of length and width is found – *not* the moiety, we notice, but the same half as in line I.32; found, moreover, as "30´ steps of 7" (a multiplication to which we shall return presently) and not through "breaking". This is "brought to" the location of the sides and then removed[24], leaving 11°30´; the trick eliminates the half of the length, but more than the third of the width. *How much more* could be found by standard procedures – the tables that tell ${}^1\!/\!_2$ to be 30´ also translate ${}^1\!/\!_3$ into 20´. Instead, the text refers to the visual procedure of Figure 16, where a 3×2-rectangle is constructed (possibly to be situated in the lower right corner of Figure 15, where the numbers appear already to be "inscribed"). ${}^1\!/\!_2$ of this rectangle is seen without further argument to exceed ${}^1\!/\!_3$ by 1 square of 6; removal of ${}^1\!/\!_2$ width thus leaves us with a rectangle $\sqsubset\!\sqsupset(l–10',w)$, the sum of whose sides must be 7–10´ = 6°50´ (line 18), whereas its area was already known to be 11°30´. The rest goes exactly as Figure 14, apart from the fact that the construction of the completed square is not spoken of as "holding" but as "inscription twice", and from the explicit separation of this construction from the computation of the numerical product of 3°25´ and 3°25´ – "*a* steps of *b*" is the expression used in the tables of multiplication, the term for the product of number by number (the same explicit separation of the two processes recurs elsewhere

[24] The term used for this removal, "cutting off", is grossly synonymous with "tearing out" when used as a mathematical term (see [Høyrup 1993b] for deeper analysis).

in the tablet, for instance in lines II.13–14 of the present problem).

As in TMS IX, Part C, the addition precedes the subtraction, as we should expect. The length of the reduced rectangle is found to be 3°50´, and the original length $l$ thus 3°50´+10´ = 4; $w$ is 3. Finally come a control and a tabulation of the result.

### *TMS XIII*[25]

**1.** 2 GUR 2 PI 5 BÁN of oil I have bought. From the buying of 1 *šekel* of silver,
2(GUR) 2(PI) 5 BÁN IÀ.GIŠ ŠÁM *i-na* ŠÁM 1 GÍN KÙ.BABBAR

**2.** 4 SÌLA of oil each (*šekel*) I have cut away.
4 SÌLA[TA.ÀM] IÀ.GIŠ *ak-ší-iṭ-ma*

**3.** $^{2}/_{3}$ *mina* of silver as profit I have seen. Corresponding to what
$^{2}/_{3}$ *ma-na* {20 ŠE} KÙ.BABBAR *ne-me-la a-mu-úr ki ma-ṣí*

**4.** have I bought and corresponding to what have I sold?
*a-šà-am ù ki ma-ṣí ap-šu-úr*

**5.** You, 4 SÌLA of oil posit and 40, (of the order of the) *mina*, the profit posit.
ZA.E 4 SÌLA Ì.GIŠ GAR *ù* 40 *ma-na ne-me-la* GAR

**6.** The IGI of 40 detach, 1´30´´ you see, 1´30´´ to 4 raise, 6´ you see.
IGI 40 *pu-ṭúr* 1,30 *ta-mar* 1,30 *a-na* 4 *i-ší* 6 *ta-mar*

**7.** 6´ to 12`50 raise, 1`17 you see.
6 *a-na* 12,50 IÀ.GIŠ *i-ší-ma* 1,17 *ta-mar*

**8.** $^{1}/_{2}$ of 4 break, 2 you see, make hold, 4 you see.
$^{1}/_{2}$ 4 *ḫi-pi* 2 *ta-mar* 2 NIGIN 4 *ta-mar*

**9.** 4 to 1`17 append, 1`21 you see. What is made equilateral? 9 is made equilateral.
4 *a-na* 1,17 DAḪ 1,21 *ta-mar mi-na* ÍB.SI 9 ÍB.SI

**10.** 9 the counterpart posit. $^{1}/_{2}$ of 4 which you have cut away break, 2 you see.
9 GABA GAR $^{1}/_{2}$ 4 *šà ta-ak-ší-ṭú ḫi-pi* 2 *ta-mar*

[25] Transliteration [TMS, 82]. Corrections, together with a translation and analysis based on the arithmetical interpretation in [Gundlach & von Soden 1963: 260–263].

**11.** 2 to the 1st 9 append, 11 you see; from the 2nd tear out,
2 *a-na* 9 1-KAM DAḪ 11 *ta-mar i-na* 9 2-KAM ZI

**12.** 7 you see. 11 SÌLA each (*šekel*) you have bought, 7 SÌLA you have sold.
7 *ta-mar* 11 SÌLA *ta-àm ta-šà-am* 7 SÌLA *ta-ap-šu-úr*

**13.** Silver corresponding to what? What shall I posit to 11 ⁽ᶜSÌLA[?]
KÙ.BABBAR *ki ma-sí mi-na a-na* 11 [ᶜsìla[?] *lu-uš-ku*]*-un*

**14.** which 12`50 of oil gives me? Posit 1,10, 1 *mina* 10 *šekel* of silver.
*šà* 12,50 Ì.GIŠ *i-na-ad-di-na* 1,[10 GAR 1 *m*]*a-na* 10 GÍN K[Ù.BABBAR]

**15.** By 7 SÌLA each (*šekel*) which you sell of oil,
*i-na* 7 SÌLA[TA.ÀM] *šà ta-pa-aš-*[*šà-ru* IÀ.GIŠ]

**16.** that of 40 of silver corresponding to what? 40 to 7 raise,
*šà* 40 KÙ.BABBAR *ki ma-ṣí* 40 *a-na* 7 [*i-ší*]

**17.** 4`40 you see, 4`40 of oil.
4,40 *ta-mar* 4,40 Í.GIŠ

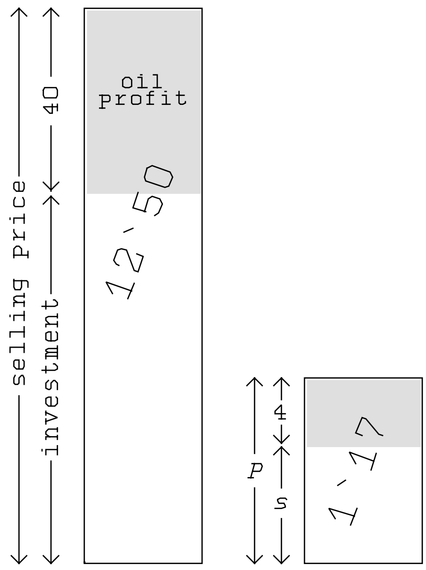


**Figure 17**.

This is another Susa text, but it certainly does not belong to the didactical genre. The problem itself recurs in less complete texts from the Babylonian core area[26].

An extra reason that the problem is perplexing is that it refers to commercial practices which are rather different from ours. A merchant has bought 2 GUR 2 PI 5 BÁN of fine vegetable oil, which later occurs as $M$ = 12`50 [SÌLA] (as we remember, 1 SÌLA is the standard unit of hollow measure, cf. p. 13), at a rate of (say) $p$ SÌLA per *šekel*. Selling at the rate of $s = p–4$ SÌLA per *šekel* he realizes a profit of $^2/_3$ *mina* or $\Pi$ = 40 [*šekel*] of

[26] A strictly parallel problem is YBC 4698 N° 9 (transliteration [MKT III, 42], cf. [Friberg 1982: 57]), which however does not tell the procedure. Related is MLC 1842 [MCT, 106], in which identical quantities of grain are bought at two different rates, and the sum of the rates and the total investment are revealed. The tablet is heavily damaged but still allows us to see that the same method was used (*mutatis mutandis*) as in the Susa text.

silver.

Lines 7–12 find $p$ and $s$ from what must have been the relations

$$p-s = 4\ ,\quad p\cdot s = 1`17\ ,$$

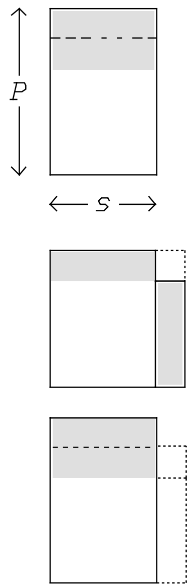

**Figure 18**.

where 1`17 = ${}^{4}/_{\Pi}\cdot M$. That $p\cdot s = {}^{4}/_{\Pi}\cdot M$ follows easily from the equation ${}^{M}/_{s}-{}^{M}/_{p} = \Pi$ if we allow ourselves some algebraic manipulation (multiplication by $ps$). This was hardly the argument from which the Babylonian calculators derived their equation, however. Firstly, this kind of symbolic manipulation was not available to them; secondly, even if they were ableto master it mentally, it would not lead to the order of operations actually found in the text but to the sequence $(M\cdot 4)\cdot\Pi^{-1}$.

Most likely, some geometric argument is in play: from line 7 onward, the procedure *is* geometric, and no jump or change of style is visible between line 6 and line 7. Moreover, since the original investment and the profit in oil are calculated in the final section of the text without having been asked for, these entities must be presumed to have played a role. This leads to the following considerations:

The total quantity of oil is the product of the selling price Σ (original investment plus profit) and the selling rate *s* (the number of SÌLA sold per *šekel*). This product we may represent by a rectangle ⊏⊐(Σ,*s*)as done in Figure 17, whose total area is 12`50 [SÌLA], and of which the part representing the profit makes up the same fraction as 4 SÌLA of the rate of purchase *p* – indeed, from what is bought for each *šekel* (i.e., *p*), 4 SÌLA is cut away as profit. A scaling operation along the vertical dimension which reduces the 40 [*šekel*] to 4 [SÌLA] will hence reduce the selling price to *p*, thus changing ⊏⊐(Σ,*s*)] into ⊏⊐(*p*,*s*). The scaling factor will have to be $4\cdot 40^{-1}$ = 6´, as found in line 6, which reduces the area of the rectangle from 12`50 to 1`17 and the original investment to *s*.

We have thus produced the starting point for the transformations of Figure 18, a rectangle with unknown sides *p* and *s* but with given area and given excess of *p* over *s*. The rest of the solution goes by the usual cut-and-paste operations. The only deviation from norms (which may have to do with the use of geometry as representation for oil and prices, but may also have other explanations) is the duplication of the breaking process in line 10 – which by the way allows the addition to precede the subtraction.

### *BM 85194 rev. II.7–21*[27]

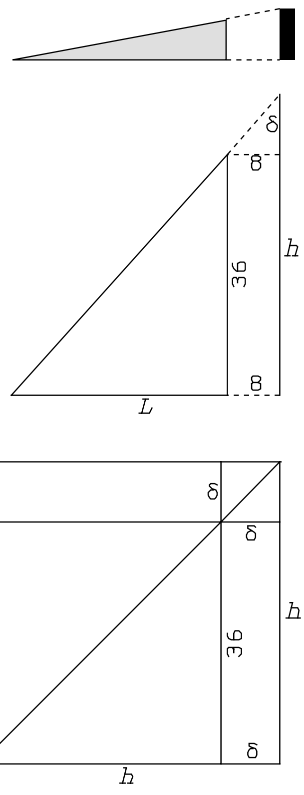


**Figure 19**.

**7.** Of dirt, 1``30` (SAR). A city inimical to Marduk I shall seize.
*i-na* SAḪAR.ḪI.A 1,30 IKU URU.KI *na-ki-i*[*r* $^{d}$MAR]DUK *a-sa-ba-at*

**8.** 6 (nindan) the (breadth of the) fundament of the dirt. 8 (NINDAN) should still be made firm before the city wall is reached.
6 ÚR SAḪAR.ḪI.A *ú-ki-in* [8 *a-n*]*a* BÀD *la sà-na-qám*

**9.** 36 (kùš) the peak (so far attained) of the dirt. How great a length
36 *zi-iq*-[*pu-um ša* SAḪ]AR.ḪI.A *ki ma-ṣí* UŠ

**10.** must I stamp in order to seize the city? And the length behind
*lu-uk-b*[*u-ús* URU.K]I *lu-uṣ-ba-at ù* UŠ EGIR

**11.** the *ḫurḫurum* (the vertical back front reached so far?) is what? You, the igi of 6, the fundament of the dirt, detach – 10´ you see. 10´ to
*ḫur-ḫ*[*u-ri* EN.NAM ZA.E IGI] 6 SUḪUS SAḪAR.ḪI.A DU$_8$.A 10 *ta-mar* 10 *a-na*

**12.** 1``30` the dirt, raise – 15` you see. The igi of 8 detach – 7´30´´ you see.
[1,30 SAḪAR.ḪI.A *i-ši* 15] *ta-mar* IGI 8 DU$_8$.A 7,30 *ta-mar*

**13.** 7´30´´ to 15` raise – 1`52°30´ you see. 1`52°30´ repeat –
[7,30 *a-na* 15 *i-š*]*i* 1,52,30 *ta-mar* 1,52,30 TAB.BA

**14.** 3`45 you see. 3`45 to 36 raise – 2``15` you see. 1`52°30´
[3,45 *ta-mar*] 3,44⟨+1⟩ *a-na* 36 *i-ši* 2,15 *ta-mar* 1,52,20⟨+10⟩

**15.** make hold – 3``30`56°15´ you see. 2``15` from 3``30`56°15´
[NIGIN 3,30,]56,15 *ta-mar* 2,15 *i-na* 3,30,56,15

[27] Transliteration [MKT I, 149].

**16.** tear out – 1``15`56°15´. What is made equilateral? 1`7°30´ you see.
[BA.ZI 1,1]5,56,14⟨+1⟩ EN.NAM ÍB.SI$_8$ 1,7,30 *ta-mar*

**17.** 1`7°30´ from 1`52°30´ tear out – 45 you see, the elevation of the city wall.
[1,7,30 *i-na*] ⌞1,5⌟2,30 BA.ZI 45 *ta-mar* SUKUD BÀD

**18.** ½ of 45 break – 22°30´ you see. The igi of 22;30 detach – 2´40´´.
[½ 45 *ḫi-pí* 2]2,[30] *ta*-[*ma*]*r* IGI 22,30 DU$_8$.A 2,40

**19.** 15` to 2´40´´ raise – 40, the length. Turn back, see 1``30`, the dirt. 22°30´,
[15 *a-na*] 2,40 *i-ši* 40 UŠ NIGÍN.NA 1,30 SAḪAR.ḪI.A *a-mur* 22,30

**20.** ½ of the elevation, to 40, the length, raise – 15` you see. 15` to 6 raise –
[½ SUKU]D *a-na* 40 UŠ *i-ši* 15 *ta-mar* 15 *a-na* 6 *i-ši*

**21.** 1``30` you see, 1``30` is the dirt. The having-been-made.
1,30 *ta-mar* 1,30 SAḪAR.ḪI.A *ne-pé-šum*

The problem is a highly artificial piece of fortification computation. A siege ramp is going to contain 1``30` [SAR] of dirt. So far, a height of 36 [kùš] has been attained, and 8 [NINDAN] still have to be completed before the city-wall is attained. The width is 6 [NINDAN]. The units call for a commentary. As we remember (see note 7), the basic measure for horizontal distance was the NINDAN (≈ 6 m), whereas that for vertical distance was the KÙŠ (= $^{1}/_{12}$ NINDAN) or cubit. Areas were measured in SAR, i.e., NINDAN$^2$, and so were volumes – for which purpose the area 1 SAR was understood as provided with a standard height 1 KÙŠ. In modern terms, the volume measure 1 SAR is thus 1 NINDAN$^2$·KÙŠ.

The text begins by eliminating the width, multiplying the volume with its reciprocal. This leads us to the two-dimensional problem shown in Figure 19 – and since the present text as well as a couple of related problems do not take into account the difference between vertical and horizontal units (as we shall see, a rectangle 1[KÙŠ]×1[NINDAN] is treated as if it were a square) we may pass immediately from the real cross-section (above) to the second diagram.

Line 12 thus finds the area of the triangle with sides *L* and *h* to be 15` – and then the procedure becomes opaque. So much is immediately clear, however, that the division by 8 in lines 12–13 finds the area of the triangle when submitted to a scaling which reduces the still missing length from 8 to 1, and that the "repetition" in line 13 completes this triangle as a rectangle: repetition – the last of the four "multiplications" – is indeed a concrete operation which provides a copy (or, if "repetition until [a small integer] $n$" is spoken of, $n$–1 copies) of an entity and joins it/them to the original; it occurs time and again in the corpus when a right triangle is completed as a rectangle.

Happily, however, two simpler problems dealing with exactly the same configuration (one found elsewhere in the present tablet, rev. II.22–23, one in BM 85210, obv. II.15–27) indicate which techniques were used. These are, firstly, a scaling which transforms the rectangle into a square with side $h$ (actually, a pseudo-square $h$[KUŠ]×$h$[NINDAN], and secondly, a comparison of areas made possible by this scaling.

The difficulty in the present case is that the scaling factor involves the unknown. This is why a preliminary scaling by a factor $^1/_8$ is performed – really an independent operation, since it precedes the "repetition". The resulting area (of the rectangle ⊏⊐($^L/_8$,$h$) is 3`45 (line 14). A supplementary scaling by a factor $\delta = h$–36 will transform this rectangle into a square □($h$); at the same time, it will change the area 3`45 into an area 3`45·($h$–36). All in all we have thus found that

□($h$) = 3`45$h$–3`45·36 .

This is the equation that is solved by standard methods in lines 14 to 19 (cf. Figure 14). At first 3`45·36 is calculated (2``15`). The bisection of the coefficient of $h$ is omitted, since 3`45 is remembered to have resulted from a doubling (the "repetition"; not just a "multiplication by 2" – "breaking" and "repetition [until 2]" are really inverse operations).

The equation is of the type that possesses two (positive) solutions – the text uses $h$ = 40, the other possibility is $h$ = 3``. However, this ambiguity does not present itself to the calculator, since it is decided already when the figure is laid out whether $h$ has to be larger or smaller than 3`45/2 – and this decision did not present itself as one in the Babylonian context, since all problems were constructed backwards from known solutions.

We should hence not wonder that the Babylonians were not aware of the double solution of this kind of equation[28]. They may well have been, but their methods and habits would always lead them unambiguously to one of the two.

### *TMS VIII, N° 1[29]*

**1.** The surface 10´. The 4th of the width to the width I have appended, to 3 I have gone ... over
[A.ŠÀ 10 4-*at* SAG *a-na* SAG DAḪ] *a-na* 3 *a-li-ik* ... ... ...[ugu]

**2.** the length 5´ goes beyond. You, 4, of the fourth, as much as width posit. The fourth of 4 take, 1 you see.
[UŠ 5 DIR]IG ZA.E [4 *r*]*e-ba-ti ki-ma* SAG GAR *re-b*[*a-at* 4 *le-qé* 1 *ta-mar*]

**3.** 1 to 3 go, 3 you see. 4 fourths of the width to 3 append, 7 you see.
[1 *a-na*] 3 *a-li-ik* 3 *ta-mar* 4 *re-ba-at* SAG *a-na* 3 D[AḪ 7 *ta-mar*]

**4.** 7 as much as length posit. 5´ the going-beyond to the tearing-out of the length posit. 7, of the length, to 4 raise,
⌈7⌉ *ki-ma* UŠ GAR 5 dirig *a-na na-sí-iḫ* UŠ GAR 7 UŠ *a-na* 4 [*i-ší*]

**5.** 28 you see, 28 the surfaces. 28 to 10´ the surface raise, 4°40´ you see.
28 *ta-mar* 28 A.ŠÀ 28 *a-na* 10 A.ŠÀ *i-ší* 4,40 *ta-mar*

**6.** 5´, the tearing-out of the length to four, of the width, raise, 20´ you see. ½ break, 10´ you see. Make hold,
⌊5⌋ *na-sí-iḫ* UŠ *a-na* 4 SAG *i-ší* 20 *ta-mar* ½ *ḫe-pe* 10 *ta-mar* NIGIN

**7.** 1´40´´ you see. 1´40´´ to 4°40´ append, 4°41´40´´ you see. What is made equilateral? 2°10´ you see.
[1,40] *ta-mar* 1,40 *a-na* 4,40 DAḪ 4,41,40 *ta-mar mi-na* ÍB.SI 2,10 *ta-ma*[*r*]

**8.** 10´ the equal (?) to 2°10´ append, 2°20´ you see. What to 28, of the surfaces, shall I posit which 2°20´ gives me?

---

[28] Which is anyhow quite rare in the corpus, but occurs in one tablet in undisguised form: TMS V, section 11b, transliteration [TMS, 44]. Of course, the corresponding rectangular problem, ▭(*l*,*w*) = *A*, *l*+*w* = *B*, is quite common (see for instance above, TMS IX, and AO 8862 N° 2) – but here, also as a matter of course, the existence of two solutions (one value for *l* and one for *w*) *was* recognized. TMS IX, moreover, by choosing for the modified length Λ the smaller number (4°30´) and for the modified width Ω the larger (28), shows awareness of the arbitrariness of the assignment of values to unknowns (Ω = 4°30´ would indeed give rise to an irregular ω and, if that were accepted, to a negative *w*).

[29] Transliteration [TMS, 52f]. Corrections, translation and discussion [Høyrup 1993a: 254–259].

[10 ¿S]Á!.SÁ? *a-na* 2,10 DAḪ 2,20 *ta-mar mi-na a-na* 28 A.ŠÀ GAR *šà* 2,20 *i-na*-[*di-n*]*a*

**9.** 5´ posit. 5´ to 7 raise, 35´ you see. 5´, the tearing-out of the length from 35´ tear out,
[5 GAR] 5 *a-na* 7 *i-ší* 35 *ta-mar* 5 *na-sí-iḫ* UŠ *i-na* 35 ZI

**10.** 30´ you see, 30´ the length. 5´ the length[30] to 4 of the width raise, 20´ you see, 20 the length (mistake for width).
[30 *ta-*]*mar* 30 UŠ 5 UŠ *a-na* 4 SAG *i-ší* 20 *ta-mar* 20 {UŠ} ⟨SAG⟩

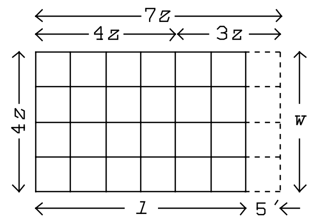

**Figure 20**.

This Susa text brings us halfway back to the didactical genre, and demonstrates another way to solve a problem similar to that of TMS IX / C, *viz* via reduction to a problem with one unknown.

The topic is the standard rectangle $\sqsubset\!\sqsupset(30',20')$. Lines 1–2 tell that

$$\sqsubset\!\sqsupset(l,w) = 10' \ , \quad w+3\cdot\tfrac{1}{4}w = l+5' \ ,$$

after which we are instructed to "posit" 4 and 7 "as much as" length and width, respectively. What takes place is a sub-division into smaller squares – see Figure 20[31]; 4 and 7, then, are not coefficients of the original length and width, as on p. 6, but the "coefficients" to the (side of the) small square which *produce* the length and the width. The total number of such squares is found by "raising" one of these "coefficients" to the other, which shows that we are really supposed to *compute* the number of small squares, not to construct a rectangle $\sqsubset\!\sqsupset(7,4)$ (in which case we should have made 4 and 7 "hold each other"). If $z$ designates the side of the small square we thus know that

$$28\square(z)-n\cdot z = 10' \ ,$$

where $n\cdot z$ represents the excess of the 28 squares over the original

30 This probably refers to the "length" of the square $\square(\frac{y}{4})$. Several other mathematical Susa texts (Nos V and VI), indeed, speak about the "length" of a square.

31 Such subdivisions are also made in a number of other texts though not with precisely the present use: BM 8390 ([MKT I, 335–337], cf. [Høyrup 1990a: 281–285]); BM 13901 Nos 10 and 11 ([MKT III, 2f], cf. [Høyrup 1990a: 278–280]).

rectangle (the rectangle contained by the "tearing-out of the length"[32] and the width) expressed in terms of the side of the small square. Only line 5 is going to compute $n = 4\cdot5' = 20'$[33]; in the meantime the equation is normalized through a multiplication by 28, whence

$$\square(28z)-n\cdot(28z) = 28\cdot10' = 4°40' \,.$$

This equation is solved by the standard procedure corresponding to Figure 8, the only deviation being the term used for "that which you have made hold" or "the made-hold" (as the entity was designated in other texts). Then $z$ is found from $28z$ as usually done when the divisor is irregular (lines 8–9), and finally $l$ and $w$ from the initial conditions, $l = 7z-5'$, $w = 4z$.

***BM 13901 N° 12***[34]

**27.** The surfaces of my two confrontations I have accumulated: 21´40´´.
A.ŠÀ *ši-ta mi-it-ḫa⟨-ra⟩-ti-ia ak-mur-ma* 21,40

**28.** My confrontations I have made hold each other: 10´.
*mi-it-ḫa-ra-ti-ia uš-ta-ki-il*$_5$*-ma* 10

**29.** The moiety of 21´40´´ you break: 10´50´´ and 10´50´´ you make hold each other,
*ba-ma-at* 21,40 *te-ḫe-pe-ma* 10,50 *ù* 10,50 *tu-uš-ta-kal*

**30.** 1´57´´21´´´40´´´´ is it. 10´ and 10´ you make hold each other, 1´40´´
1,57,21{+25},40[35]-E 10 *ù* 10 *tu-uš-ta-kal* 1,40

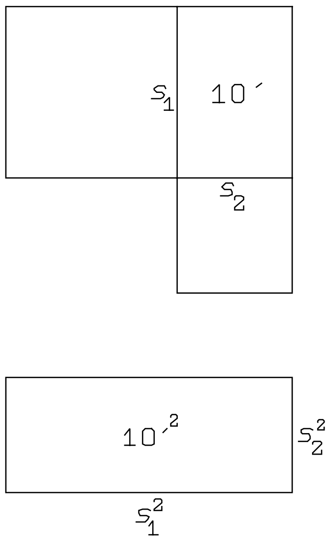


**Figure 21.**

[32] This entity occurred already in TMS VII, we remember – cf. p. 10. Once again we may observe that the use of the term for "that which should be torn out from $7z$ in order to produce the (real) length" implies that $7z$ is itself regarded as *a length*.

[33] This delayed computation of the first-degree "coefficient" is habitual, cf. BM 13901 N° 14, rev. I.2–3.

[34] Transliteration [MKT III, 3].

[35] Erroneous for 1,57,21,40.

**31.** inside 1´57´´21´´´40´´´´ you tear out: 17´´21´´´40´´´´ makes 4´10´´ equilateral.
*lìb-bi* 1,57,21{+25},40 *ta-na-sà-aḫ-ma* 17,21{+25},40[36]-E 4,10[37] ÍB.SI$_8$

**32.** 4´10´´ to one 10´50´´ you append: 15´ makes 30´ equilateral.
4,10 *a-na* 10,50 *iš-te-en tu-ṣa-ab-ma* 15-E 30 ÍB.SI$_8$

**33.** 30´ the first confrontation.
30 *mi-it-ḫar-tum iš-ti-a-at*

**34.** 4´10´´ inside the second 10´50´´ you tear out: 6´40´´ makes 20´ equilateral.
4,10 *lìb-bi* 10,50 *ša-ni-im ta-na-sà-aḫ-ma* 6,40-E 20 ÍB.SI$_8$

**35.** 20´ the second confrontation.
20 *mi-it-ḫar-tum ša-ni-tum*

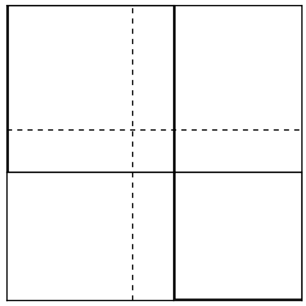

**Figure 22**.

This problem comes from the collection of "square problems" which already supplied us with three illustrations of the basic second-degree techniques. Even in the present case basic techniques are drawn upon, but in a surprising way.

The problem is alluringly simple. Two squares $\square(s_1)$ and $\square(s_2)$ are involved, and we know the sum of their areas and the rectangle held by their sides:

$$\square(s_1)+\square(s_2) = 21'40'' \ , \quad \sqsubset\!\sqsupset(s_1,s_2) = 10' \ .$$

This could have been solved by means of the diagram of Figure 22, which appears already to have served in the solution of N° 8 of the same tablet,

$$\square(s_1)+\square(s_2) = 21'40'' \ , \quad s_1+s_2 = 50' \ .$$

All that is needed is to add twice the rectangle $\sqsubset\!\sqsupset(s_1,s_2)$ to the sum of the square areas, which would reduce the problem to the second step of the solution of N° 8. The actual reduction is quite different, and quite sophisticated (see Figure 21): it *represents* the areas $\square(s_1)$ and $\square(s_2)$ by the sides of a rectangle, whose area must then be $\sqsubset\!\sqsupset(\square(s_1),\square(s_2))$, computed as

[36] Erroneous for 17,21,40—a consequence of the previous error.

[37] This number is correct but *not* the square-root of 17,46,40.

□(⊏⊐($s_1$,$s_2$)) = 10´·10´[38]. Thereby the two-square problem is reduced to one of the basic rectangular problems – one which we have already encountered in TMS IX and AO 8862 N° 2. As in these, addition is seen to precede subtraction.

According to palaeographic and other internal criteria, the tablet is one of the earliest Old Babylonian mathematical text. This makes the present problem the very first surviving instance of indubitable algebraic representation. Modest as it seems, it bears witness to an utterly consequential leap in mathematical thinking – even (given the basic importance of mathematical representation in the modern technical and scientific civilization) of one of the major moments in human intellectual history.

### *YBC 6504 N° 4*[39]

**11.** So much as length over width goes beyond, encountered, from inside the surface I have torn out:
*ma-la* UŠ U[.G]Ù SAG SI UL.UL *i-na* A.ŠÀ BA.Z[I¿-*ma*?]

**12.** 8´20´´. 20´ the width, its length what?
8,20 20 SAG UŠ.BI EN.NAM

**13.** 20´ encountered: 6´40´´ you posit.
20 UL.UL-*ma* 6,40 IN.GAR

**14.** 6´40´´ to 8´20´´ you append: 15´ you posit.
6,⌞40 *a*⌝-*na* 8,20 BÍ.DAḪ-*ma* 15 IN.GAR

**15.** 15´ makes 30´ equilateral. 30´ as length you posit.
15-e 30 ÍB.SI$_8$ 30 UŠ IN.GAR

So far we have seen how effectively the Babylonian calculators were able to use their cut-and-paste and scaling techniques; apart from a few writing errors, the only mistake we have encountered so far was the short-circuit in the end of VAT 8391 N° 3.

The first three problems of the present tablet are solved just as correctly, and only differ in style from what we have seen so far by a heavy use

[38] That ⊏⊐(□(AB),□(BΓ) equals □(⊏⊐(AB,BΓ)) (for numbers *a* and *b*, but in geometrical terminology) is discussed and proved by Hero in his proof of the so-called “Hero’s formula” (*Metrika* I.7, ed. Schöne 1903: 16–18 – somewhat distorted in the German translation).

[39] Transliteration [MKT III, 23], analysis [Høyrup 1989: 30f].

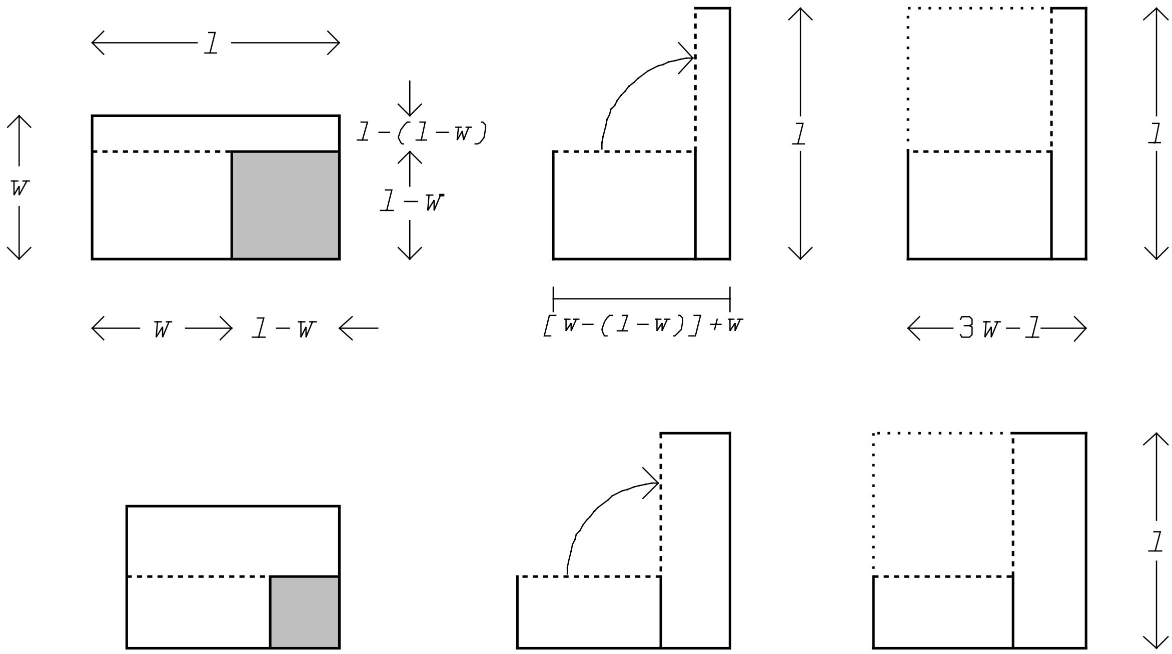


**Figure 23**.

of Sumerograms and by a slightly deviating terminology: even the end result is "posited"; and the construction of a square is not spoken of as a confrontation of equals but with a comparable metaphor (referring to an encounter in battle).

The solution of problem 4, however, is mistaken though numerically correct, and probably the outcome of too rash reliance on the visually obvious. Figure 23 shows what has probably been the manoeuvre – above in distorted proportions, where the fallacy is obvious, below in correct measures, which masks the mistake and thus makes it understandable:

From a rectangle, the square on the excess of the length over width is removed; the remaining area is told, as is the width itself:

$$\sqsubset\!\sqsupset(l,w)-\square(l-w) = 8'20'' , \quad w = 20' .$$

The rectangle seems to be opened so as to allow the completion by $\square(w)$, and the completed rectangle is taken to be a square with side $l$. Actually, it is a rectangle $\square(l,3w-l)$, and only because $l = {}^{3}/_{2}w$ do the two coincide.

Several texts betray that the oral exposition will often have identified unknown entities by their actual numerical value even when this value was not given in the statement (nor used for the solution – cf. [Høyrup 1992: 354f]). It was thus normal that teacher as well as student knew the answer in advance. Besides, as we have seen, rectangle problems almost invariably dealt with ⊏⊐(30´,20´) or ⊏⊐(30,20); teaching will have aimed very explicitly, not at finding the solution but at showing *how* to derive it. Normally, however, the texts are able distinguish between values that are *given* and numbers that are simply used as names; mistakes of the present kind, induced by the possession of knowledge beyond what is supposed to be given, are quite rare. If we think of how heavily *we* rely on algebraic formalism when we try to locate the errors (e.g., what is written into Figure 23), the normally skilful distinction between what is given and what is merely known is quite impressive.

## IV. "ALGEBRA"-RELATED GEOMETRY

We shall close this presentation of Old Babylonian "algebra" by looking at a few texts which do not belong to the genre proper but use some of the techniques that we have come to know.

***IM 55357[40]***

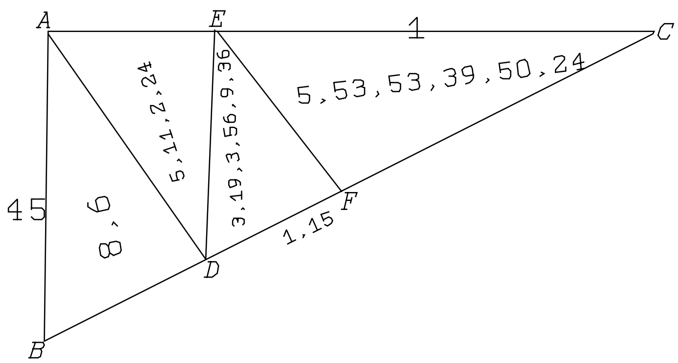


**Figure 24**.

**1.** A triangle. 1` the length, 1`15 the long length, 45 the upper width.
SAG.DÙ 1 UŠ 1,15 UŠ GÍD 45 SAG.KI AN.TA

**2.** 22`30 the complete surface. From 22`30 the complete surface, 8`6 the upper surface.

40 Transliteration [Baqir 1950].

22,30 A.ŠÀ TIL *i-na* 22,30 A.ŠÀ TIL 8,6 A.ŠÀ AN.TA

**3.** 5`11°2´24´´ the adjacent surface, 3`19°3´56´´9´´´36´´´´ the 3d surface.
5,11,2,24 A.ŠÀ TA 3,19,3,56,9,36 A.ŠÀ 3-KÁM

**4.** 5`53°53´39´´50´´´24´´´´ the lower surface.
5,53,53,39,50,24 A.ŠÀ KI.TA

**5.** The upper length, the shoulder length, the lower length and the descendant what?
UŠ AN.TA UŠ.MÚRGU UŠ.KI.TA *ù mu-tar-ri-it-tum mi-nu-um*

**6.** You, in order to know the making, the IGI of 1` the length detach, to 45 raise,
ZA.E AK.TA.ZU.UN.DÈ IGI 1 UŠ DU$_8$.A *a-na* 45 ÍL

**7.** 45´ you see. 45´ to 2 raise, 1°30´ you see, to 8`6 the upper surface
45 IGI.DÙ 45 NAM 2 ÍL 1,30 IGI.DÙ 1,30 NAM 8,6 A.ŠÀ AN.TA

**8.** raise, 12`9 you see. 12`9 makes what equilateral? 27 the equilateral.
ÍL 12,⸤9⸥ IGI.DÙ 12,9 A.BA.ÀM ÍB.SI$_8$ 27 ÍB.SI$_8$

**9.** 27 the width. 27 break, 13°30´ you see. The igi of 13°30´ detach,
27 SAG ¿(erasure)? 27 *ḫi-pí* 13,30 IGI.DÙ IGI 13,30 DU$_8$.A

**10.** to 8`6 the upper surface raise, 36 you see, the length (which is) counterpart of the length 45, the width.
NAM 8,6 [A.Š]A AN.TA ÍL 36 IGI.DÙ UŠ GABA UŠ 45 SAG.KI

**11.** Turn around. The length 27, of the upper triangle, from 1`15 tear out,
*na-ás-ḫi-ir* UŠ 27 SAG.DÙ AN.TA *i-na* 1,15 BA.ZI

**12.** 48 leave. The IGI of 48 detach, 1´15´´ you see, 1´15´´ to 36 raise,
48 ÍB.TAG$_4$.A IGI 48 DU$_8$.A 1,15 IGI.DÙ 1,15 NAM 36 ÍL

**13.** 45´ you see. 45´ to 2 raise, 1°30´ you see, to 5`11°2´24´´ raise,
45 IGI.DÙ 45 NAM 2 ÍL 1,30 IGI.DÙ 1,30 NAM 5,11,2,24 ÍL

**14.** 7`46°33´36´´ you see. 7`46°33´36´´ makes what equilateral?
7,46,33,36 IGI.DÙ 7,46,33,36 A.BA.ÀM ÍB.SI$_8$

**15.** 21°36´ the equilateral, the width of the 2nd triangle.
21,36 ÍB.SI$_8$ 21,36 SAG.KI ⟨SAG⟩.DÙ 2-KÁM

**16.** The moiety of 21°36´ break, 10°48´ you see. The IGI of 10°48´ detach,
BA 21,36 ⟨*hi*⟩-*pí* 10,48 IGI.DÙ IGI 10,48 DU$_8$.A

**17.** to ⟨...⟩
NAM

The diagram in Figure 24 reproduces the features of a drawing on the tablet as faithfully as possible, including the location and orientation of numbers (the letters are evidently added). The numbers show that $\Delta ABC$ is right, and that the others are cut off by successive heights. All triangles are thus right, and all are similar to the 3-4-5 triangle.

This explanation probably goes beyond what the Babylonian calculators would have told. They appear to have possessed no concept of quantified angle, and so to speak to have distinguished a "right" from a "wrong" angle – just as the present text distinguishes the length *simpliciter*, i.e., the genuine length, the length that serves when the area is computed, from the "long length", the hypotenuse $BC$. The way the successive triangles are named and the successive steps of the solution suggests an intuitive idea that cutting off a triangle at a "good" angle would give you a new triangle "of the same kind" as the original one, i.e., with the same ratio between the sides ("similar" even in our technical sense). We are not informed about how the successive areas of the statement were computed, but from what we know about scaling in one and two dimensions and from the computations that follow later in the text it is most likely that the "upper surface" $\Delta ABD$ was computed as $({}^3/_5)^2$ times the "complete surface" $\Delta ABC$ – the "adjacent surface" $\Delta ADE$ as $({}^3/_5)^2$ times the area $\Delta ADC$ (itself found by subtraction of $\Delta ABD$ from $\Delta ABC$ – the "third surface" $\Delta EDF$ through another repetition of this subtraction followed by a multiplication by $({}^3/_5)^2$ – and the "lower surface" finally by subtraction alone. It is also possible though not in harmony with the steps of the subsequent procedure that $\Delta ADE$ was found as $({}^4/_5)^2$ times $\Delta ABD$, etc. Unlikely, on the other hand, is a direct computation of the successive heights (those asked for in the text) and the corresponding bases – but even this possibility cannot be fully ruled out since only the numbers are there.

Equally undecidable in principle is the identification of the "upper length", the "shoulder length", the "lower length" and the "descendant" – but it would be strange if the lengths were not the lengths (*simpliciter*) of the corresponding partial triangles ($AD$, $DE$ and $EF$), and the "descendant" the segment *FC*.

The solution makes use of that doubling of a right triangle into a rectangle which we encountered in connection with the siege ramp, and of the scaling in one dimension which transforms a rectangle into a square. At first, indeed, the text calculates the ratio between the length and the width of Δ*ABC* (and clearly presupposes it to hold for Δ*ABD*). Multiplying the area 8`6 by twice this ratio gives us the area of the square on the width *BD*, from which *BD* itself is found in line 8[41]. The height *AD* is then found (line 10) from the area and the moiety of *BD*.

Line 11 finds *DC* as 1`15–*BD* = 48, and uses this to find the scaling ratio which transforms the doubled Δ*ADC* (and, it is obviously assumed, the doubled Δ*ADE*) into a square on its width. Continuing as before, *AE* is found in line 15 to be 21°36´. The text breaks off before *DE* is found, but it is obvious how the computation would have to go on.

### *VAT 8512*[42]

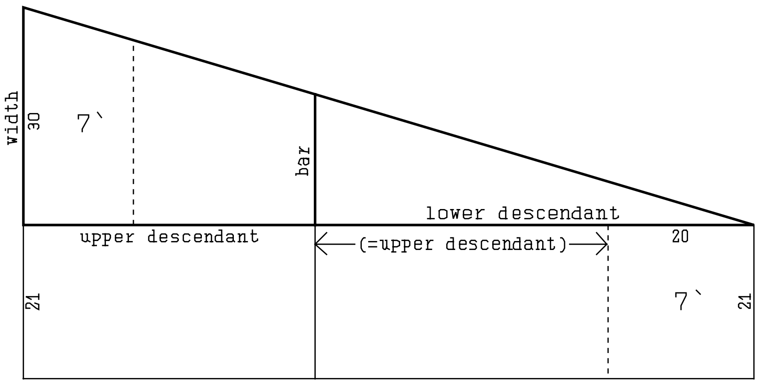


**Figure 25**.

**Obverse**

**1.** A triangle. 30 the width. In the inside two plots,
[¿SAG.DÚ 30 SAG *i-na li-ib-bi ši-it-ta*? *t*]*a-wi-ra-tum*

**2.** the upper surface over the lower surface 7` goes beyond.
[¿...? A.ŠÀ AN.TA U.GÙ A.ŠÀ] KI.TA 7 *i-tir*

**3.** The lower descendant over the upper descendant 20 goes beyond.
*m*[*u-tar-ri-tum* KI.TA U.GÙ *mu-tar-ri-tim*] AN.TA 20 *i-tir*

**4.** The descendants and the bar what?

---

41 Since it is the ratio that is multiplied and not the area, “raising” is used instead of “repetition”. The latter operation could only come in play if the area were multiplied *first* by 45´, thus producing an isosceles rectangle.

42 Transliteration [MKT I, 341f], cf. [TMB, 101–103] and [von Soden 1939: 148].

*mu-tar-ri-d*[*a*]-⌈*tum ù pi-i-i*]*r-kum mi-nu-*[*u*]*m*

**5.** And the surfaces of the two plots what?
*ù a-š*[*a*] *ši-it*[¿*-ta ta-wi*?]*-ra-tum mi-nu-u*[*m*]

**6.** You, 30 the width posit, 7` which the upper surface over the lower surface goes beyond posit,
*at-ta* 30 SAG GAR.RA 7 *ša* A.ŠÀ AN.TA U.GÙ A.ŠÀ KI.TA *i-te-ru* GAR.RA

**7.** and 20 which the lower descendant over the upper descendant goes beyond posit.
*ù* 20 *ša mu-tar-ri-t*[*um* K]I.TA U.GÙ *mu-tar-ri-tim* AN.TA *i-te-ru* G[AR.R]A

**8.** The IGI of 20 which the lower descendant over the upper descendant goes beyond
IGI 20 *ša mu-tar-ri-tum* KI.TA U.GÙ *mu-tar-ri-tim* AN.TA *i-te-ru*

**9.** detach: 3´ to 7` which the upper surface over the lower surface goes beyond
*pu-ṭur-ma* 3 *a-na* 7 *ša* A.ŠÀ AN.TA U.GÙ A.ŠÀ KI.TA *i-te-ru*

**10.** raise, 21 may your head retain!
ÍL 21 *re-eš-ka li-ki-il*

**11.** 21 to 30 the width append: 51
21 *a-na* 30 SAG *ṣi-ip-ma* 51

**12.** together with 51 make hold: 43`21
*it-ti* 51 *šu-ta-ki-il-ma* 43,21

**13.** 21 which your head retains together with 21
21 *ša re-eš-ka ú-ka-lu it-ti* 21

**14.** make hold: 7`21 to 43`21 append: 50`42.
*šu-ta-ki-il-ma* 7,21 *a-na* 43,21 *ṣi-ip-ma* 50,42

**15.** 50`42 to two break: 25`21.
50,42 *a-na ši-na ḫi-pí-ma* 25,21

**16.** The equilateral of 25`21 what? 39.
ÍB.SI$_8$ 25,21 *mi-nu-um* 39

**17.** From 39, 21 the made-hold tear out, 18.
*i-na* 39 21 *ta-ki-il-tam ú-sú-uḫ-ma* 18

**18.** 18 which you have left is the bar.
18 *ša te-zi-bu pi-ir-kum*

**19.** Well, if 18 is the bar,
*ma šum-ma* 18 *pi-ir-kum*

**20.** the descendants and the surfaces of the two plots what?
*mu-tar-ri-da-tum ù* A.ŠÀ *ši-i*[*t-ta ta-wi-ra-tim mi-nu-um*]

**21.** You, 21 which together with itself you have made hold, from 51
*at-ta* 21 *ša a-na r*[*a-ma-ni-šu tu-uš-ta-ki-lu i-na* 51]

**22.** tear out: 30 you leave. 30 which you have left
*ú-sú-uḫ-ma* 30 *te-z*[*i-ib* 30 *ša te-zi-bu*]

**23.** to two break, 15 to 30 which you have left raise,
*a-na ši-na ḫi-pí-ma* 1[5 *a-na* 30 *ša te-zi-bu* ÍL]

**24.** 7`30 may you head retain!
7,30 *re-eš*[*-ka li-ki-il*]

**Edge**

**1.** 18 the bar together with 18 make hold:
18 *pi-i*[*r-kam it-ti* 18 *šu-ta-ki-il-ma*]

**2.** 5`24 from 7`30 which your head retains
5,24 [*i-na* 7,30 *ša re-eš-ka ú-ka-lu*]

**3.** tear out: 2`6 you leave.
*ú-sú-*[*u*]*ḫ-ma 2,6 te-*[*zi-ib*]

**Reverse**

**1.** What to 2`6 shall I posit
*mi-nam a-na* 2,6 *lu-uš*[*-ku-un*]

**2.** which 7` which the upper surface over the lower surface goes beyond gives me?
*ša* 7 *ša* A.ŠÀ [AN.TA U.GÙ] A.ŠÀ KI.TA *i-*[*te-ru*] *i-na-di-nam*

**3.** 3°20´ posit. 3°20´ to 2`6 raise, 7` it gives you.
3,20 GAR.RA 3,20 *a-na* 2,6 ÍL 7 *it-ta-di-kum*

**4.** 30 the width over 18 the bar what goes beyond? 12 goes beyond.
30 SAG U.GÙ 18 *pi-ir-ki mi-nam i-tir* 12 *i-tir*

**5.** 12 to 3°20´ which you have posited raise, 40.
12 *a-na* 3,20 *ša ta-aš-ku-nu i-ši* 40

**6.** 40 the upper descendant.
40 *mu-tar-ri-tum* AN.TA

**7.** Well, if 40 is the upper descendant,
*ma šum-ma* 40 *mu-tar-ri-tum* AN.TA

**8.** the upper surface is what? You, 30 the width,
A.ŠÀ AN.TA *mi-nu-um at-ta* 30 SAG

**9.** 18 the bar accumulate: 48 to two break: 24.
18 *pi-ir-kam ku-mur-ma* 48 *a-na ši-na ḫi-pí-ma* 24

**10.** 24 to 40 the upper descendant raise, 16`.
24 *a-na* 40 *mu-tar-ri-tim* AN.TA ÍL 16

**11.** 16` the upper surface. Well, if 16` the upper surface,
16 A.ŠÀ AN.TA *ma šum-ma* 16 A.ŠÀ AN.TA

**12.** the lower descendant and the lower surface what?
*mu-tar-ri-tum* KI.TA *mi-nu-um ù* A.ŠÀ KI.TA MI.NU.UM

**13.** You, 40 the upper descendant to 20 which the lower descendant over the upper descendant goes beyond
*at-ta* 40 *mu-tar-ri-tam* AN.TA *a-na* 20 *ša mu-tar-ri-tum* KI.TA U.GÙ *mu-tar-ri-tim* AN.TA *i-te-ru*

**14.** append, 1` the lower descendant.
*si-ib-ma* 1 *mu-tar-ri-tum* KI.TA

**15.** 18 the bar to two break; 9
1[8] *pi-ir-kam a-na ši-na ḫi-pí-ma* 9

**16.** to 1` the lower descendant raise, 9`.
*a-na* 1 *mu-tar-ri-tim* KI.TA ÍL 9

**17.** 9 the lower surface.
9 A.ŠÀ KI.TA

The problem deals, like VAT 8391 Nº 3, with a field that is subdivided into two "plots". There, however, the similarity between the two problems stops, and the present text is in fact an ingenious piece of pure geometry.

The field is triangular, and for convenience we may assume it to be right, in which case the "descendants" are sections of the length (*simpliciter*)[43]. A "bar" (a parallel transversal) separates the two plots from each other, and we are told the width, the difference between the partial areas, and the difference between the two partial lengths.

The computation of the "bar" makes use of an ingenious trick (first unravelled by Solomon Gandz [1948: 36f], more clearly explained by Peter Huber [1955]), belonging to the same genre as the quadratic completion. Just as the quadratic completion allows us to replace a rectangle by a square, the present completion reduces the unequal partition of the triangle to a bisection of a trapezium – a problem whose solution was known by

[43] In principle, any triangular shape would do, if only the "descendants" were sections of the height. But current habits as known from other texts support the conjecture that a "right" and no "wrong" triangle was meant.

Mesopotamian surveyors at least since the 23d millennium B.C. As shown in Figure 25, a rectangle is joined to the triangle, and its width (21) determined in lines 8–10 in such a way that the area which is adjacent to the excess of the lower over the upper descendant (20) equals the excess of the upper over the lower plot (7`). The prolonged bar will thus bisect the trapezium that results when the rectangle is joined to the triangle.

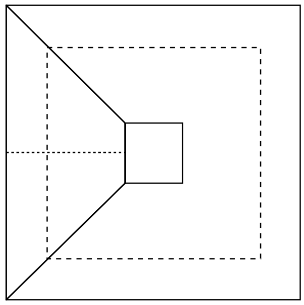

**Figure 26**.

In lines 11 to 15, the areas of the squares on the parallel sides of the trapezium are found and their average (the moiety of their sum) is computed. This average is indeed the square on the bisecting transversal, as can be easily argued from Figure 26 (whether one looks at the isosceles or one of the right trapezia – our usual scaling operation in one dimension may have to be applied). Since this transversal turns out to be 39, the original "bar" must be 18 (line 18).

What follows next is an elimination of the added rectangle, from which we recalculate the width of the triangle, and then in lines 22f the area of the isosceles right triangle on this side (7`30). The triangle has thus been submitted to a scaling operation which transforms it into a semi-square – cf. Figure 27. In the next step, the square on the bar is found (5`24), i.e., twice the area into which the lower plot is scaled or, indeed, the lower plot and as much of the upper plot as equals the lower plot. Subtraction of 5`24 from 7`30 thus leaves that which results from the scaling of the excess of the upper over the lower plot, i.e., the shaded area of Figure 27.

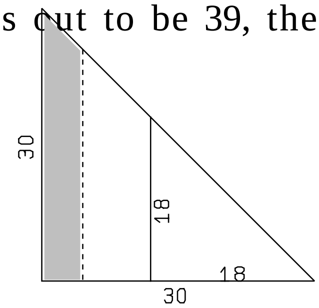


**Figure 27**.

This allows us to find the inverse scaling factor (3°20´, rev. line 3); "raising" the difference (12) between the width and the bar (which equals their distance in Figure 27) to 3°20´ gives their distance in the original triangle (40, lines 5f), whence the upper surface can be computed (16`, line 11). The lower descendant is found from the upper descendant and from the difference between the two (1`, line 14), and the lower surface finally by the usual formula for the triangular area.

Even if *we* find it convenient to distinguish the Old Babylonian algebraic genre from other kinds of geometric computation, the terminology and operations of the present text thus suggests that this is really *our* distinction – a distinction which, if not directly alien to Old Babylonian mathematical thought, was hardly felt to be of major importance.

## V. APPENDIX: RECAPITULATION OF TERMINOLOGY AND OPERATIONS

### *Additive operations*

Of these there are two. One, "appending" (*wasābum* / DAḪ), is a concrete process in which one entity is joined to another and absorbed by it. For the same reason, no separate term for the sum by this operation exists – the absorbing entity so to speak conserves its identity while increasing in magnitude, and if the operation is geometrical it stays in place.

The other additive operation, "accumulating" (*kamārum* / GAR.GAR / UL.GAR), may but need not be concretely meaningful. It can even apply to the addition of measuring numbers for entities of different dimension, and is thus the operation which allows the addition of [the measure of] sides and [the measure of] areas. In one text (AO 8862), where the operation was a concretely meaningful heaping, we have seen the sum designated by a plural, as "the things accumulated" (*kimrātum*); most often (e.g., TMS VII) it is designated by the Sumerogram UL.GAR, which allows no interpretation beyond the translation "accumulation".

### *Subtractive operations*

Even subtractions are of two kinds. One is the concrete removal of part of an entity which otherwise conserves its identity (in the sense that it remains in place if spatially located, and that no term exists for the difference except the descriptive "what remains"); the standard term is "tearing out" (*nasāḫum* / ZI), but a number of near-synonyms can be found (one, "cutting off" / *ḫarāṣum*, occurred in AO 8862) in situations where their general connotations fit the concrete process. Since one entity has to be part of the other, it can only be used when the subtraction is meaningful[44].

The other is a comparison, the result of which notices how much one entity "goes beyond" (*watārum* / DIRIG) another. Even this operation is only used when it is concretely meaningful. The difference may be designated simply by the Sumerogram DIRIG ("the going-beyond"), or by a full Akkadian phrase meaning "so much as *A* over *B* goes beyond" (used, e.g., in YBC 6504 N° 4, line 11).

### *"Multiplications"*

Four distinct operations are traditionally understood as (one and the same) multiplication ("there *is* only one", as Thureau-Dangin remarked somewhere). All occur in the texts that were discussed above.

"Raising" (*našûm* / ÍL, with the synonymous set *ullûm* / NIM), was originally a spatial metaphor used for the calculation of volumes: a prism with base *A* SAR and height *h* KÙŠ is obtained when the base, provided with the standard height 1 KÙŠ, is raised from 1 to its real height *h*. From this use, the concept was generalized to other computations involving a similar consideration of proportionality[45] – e.g., the *computation* of an area from the width and the length implicitly provided with a standard breadth (a "projection") 1 – ultimately evolving into a general concept of multiplicative computation of concrete magnitudes[46].

The tables listing the product of number by number refer to a different idea, that of repeated addition. The expression is *n* A.RÁ *a*, "*n* steps of *a*".

---

44 This claim seems to be contradicted by BM 13901 N° 2, where a side is "torn out" from the area, apparently just the way the two are "accumulated" in N° 1. N° 3 from the same tablet shows, however, that the "tearing" involves an automatic shift of conceptualization and the implicit involvement of a "projection".

45 We may remember the Euclidean definition of the product $a \cdot b$ as the number which is to *b* as *a* is to 1 – the number which, in the simple version of proportionality used in *Elements* VII, contains *b* as often as *a* contains the unit (*Elements* VII, def. 15). Basing multiplication on proportionality and not *vice versa* is thus not a Babylonian specialty.

46 This point is already reached in the earliest Old Babylonian texts at our disposal; only the observation that the order of factors is fixed when volumes are involved (invariably, it is the base which is raised to the height) but erratic in all other cases allows us to establish that the facile interpretation corresponds in fact to the original usage.

In less formal mathematical discourse, the same idea was expressed both in TMS VII – where "going *a* to 10" produced 10*a*, *a* being spoken of precisely as the "step" – and in TMS VIII, where "appending" *a* to *B* and "going to 3" brought forth *B*+3*a*.

The two remaining operations are not in themselves genuine multiplications but concrete operations which *entail* a multiplication. One is the construction of a rectangle ("building" in AO 8862), "making *l* and *w* hold each other" (*šutakūlum* / Ì.KÚ.KÚ), with a number of near-synonyms[47]. In a few texts (thus AO 8862), the concomitant computation is made explicit, mostly it is left as an implicit consequence of the construction and the result given immediately.

"Repeating" or "repeating to *n*" (*eṣēpum* / TAB), finally, is a concrete doubling or *n*-doubling. It only occurs with small integer values for *n* ($n$<10), and only when a real mirroring or an agglomeration of identical copies is involved – in the ramp problem from BM 85194 it was used when a triangle was doubled into a rectangle. Even this operation implies a multiplication of the measuring number by *n*.

### *Division*

Division was no procedure in Babylonian mathematics. It was a problem. If *d* is a sexagesimally regular number, i.e., if *d* can be expressed in the form $2^p \cdot 3^q \cdot 5^r$ and its reciprocal thus a as finite sexagesimal fraction, this reciprocal (the IGI of *d*) is "detached" (*paṭārum* / $\text{DU}_8$), actually looked up, and then raised to *A* ($A \div d = A \cdot {}^1/_d$). If the IGI cannot be looked up in the standard table of reciprocals, the text states that "its IGI I do not know", poses the question "what shall I posit to *d* which gives me *A*" and then gives the answer (at times designated *bandûm*) immediately; since all mathematical texts were constructed backwards from known results, this could always be done.

### *Bisection*

The normal or "incidental" half (*mišlum*) – that which stands on a par

[47] NIGIN, the basic meaning of which is to "surround" or "contain", was used logographically for *šutakūlum* in TMS IX and probably in other Susa texts as well.

with other fractions – was found through multiplication by 30´ (thus in AO 8862 N° 2). In cases, however, where the half could be nothing but precisely the half, it carried a particular name, the "moiety" (*bāmtum*) – and so did the process by which the moiety is found: "breaking" (*ḫepûm* / GAZ). This is the term that occurs when the width of a (right) triangle is bisected and then "raised" to the length; above, we have mostly encountered it when a rectangle was transformed into a gnomon. In both cases, a "necessary" and no "incidental" half is indeed involved.

### *Squaring and square root*

By accident, all four "multiplications" might happen to involve two identical factors. However, only cases where a geometrical squaring is meant (directly or in representation) refer to a particular concept and terminology.

The square produced in the process is a "confrontation [of equals]" (*mitḫartum*), parametrized by the side and *possessing* its area. Often, when one side is found, the side which meets it in a corner is characterized as its "counterpart" (*meḫrum* / GABA[48]). The process itself is spoken of as "making *a* confront itself" (*šutamḫurum* – derived from the same verbal root *MḪR* as *mitḫartum* and *meḫrum*; often, NIGIN functions as a logogram for this verb rather than for *sutakūlum*). Various synonyms may be used, e.g., "encountering" (UL.UL)[49].

The opposite movement, finding the side of a square area, is spoken of in one of the few genuinely Sumerian expressions that occur in our texts[50]. The full phrase is *A*-E *s* ÍB.SI$_8$ ("*A* makes *s* equilateral"), meaning that *A*, if laid out as a square area, produces *s* as its parametrizing "equal-side". Often, the term ÍB.SI$_8$ (originally a verb, and still remembered in other texts to be a verb of which *A* is the agent and *s* the object) is treated like

48 In IM 55357, line 10, the "length" of a particular right triangle was identified as "the counterpart" of the identifiable "long length" of the same triangle.

49 One may observe that the Gilgameš epic speaks of Enkidu as the *meḫrum* of Gilgameš, and about the predicted fight between the two peers in strength using the verb *šutamḫurum*.

50 Other genuinely Sumerian terms are IGI, UŠ (length) and SAG (width or "front" of a rectangle). A.ŠÀ, "surface", is always written with the Sumerogram but often provided with a phonetic complement that shows it to have been pronounced in Akkadian.

a noun, *s* being seen as “the equilateral of *A*”.

At times, the prefix ÍB is replaced by BA without any apparent change of meaning. The idea, launched in the 1930es, that $BA.SI_8$ designate the cube root, has been discredit by texts discovered since then. The preference may to some extent have been geographically determined.

### *Units*

Practical life made use of traditional measures, whose mutual relations were sexagesimally regular numbers but apart from that only slightly better adapted to the sexagesimal number system than British measures are to the decimal system. For computational purposes, the Babylonian calculators made use of metrological tables converting all measures to sexagesimal multiples of a set of basic units – the NINDAN for horizontal distances, the KÙŠ ( = $^1/_{12}$ NINDAN ≈ 50 cm) for vertical distances, the SÌLA (≈ 1 litre) for hollow measures, the SAR for areas and volumes (meaning $\text{NINDAN}^2$ and $\text{NINDAN}^2\cdot$KÙŠ, respectively). The same conversions, and the same choice of basic units, recur in the mathematical texts – and for good reasons: the students trained by the texts we know were expected to become practical calculators, and the only practical purpose of the extensive work on second-degree algebra was the drill of sexagesimal computation and conversions which it entailed[51].

## INDEXES

### *Index 1: discussions of terms and operations*

This first index locates the main discussions of the single terms and operations. The standard translations are used as key words. The second index lists Akkadian terms and Sumerograms with cross-references to the standard translations.

51 This is not the place to discuss the *social* function of second-degree algebra for professional pride, which has been similar to the function of Latin and Greek for clerks of a later age.

## Index 2: Akkadian terms and Sumerograms with standard translations

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